Operations Research Letters 49 (2021) 191–196

Contents lists available at ScienceDirect

Operations Research Letters

journal homepage: www.elsevier.com/locate/orl

Selling multiple units to strategic consumers

Chen Jin [a], Qian Liu [b,*], Chenguang (Allen) Wu [b]

[a] National University of Singapore, Singapore
[b] Hong Kong University of Science and Technology, Hong Kong

ARTICLE INFO

Article history:
Received 2 September 2020
Received in revised form 31 December 2020
Accepted 31 December 2020
Available online 12 January 2021

Keywords:
Multi-unit purchase
Strategic customer behavior
Price commitment
Market segmentation

ABSTRACT

We consider a monopolist firm selling to strategic customers who may purchase more than one unit of a product in a two-period model. We provide closed-form solutions for the firm's optimal prices and show that they are non-monotonic in both the value of the second unit and the strategic level of customers. Particularly, the first-period price can increase as customers become more strategic, in contrast to the single-unit setting where it always decreases in the strategic level of customers.

1. Introduction

Dynamic pricing is widely used to manage customer demand and improve revenue. It creates an incentive for customers to plan the timing of their purchases strategically in order to maximize individual utility. To differentiate these customers from those who purchase immediately as long as they gain nonnegative utility, the term "strategic customers" is used to refer to customers who take the entire price path into account before making purchase decisions, while "myopic customers" consider only the current price. The strategic purchasing behavior of customers has great implications on a firm's pricing decisions, of which both industrial practitioners and academic researchers are aware. For example, in the retailing industry in the United States, the weeks before Christmas are very crucial for retailers since holiday shoppers have been trained to wait until the last minute for deals. According to the National Retail Federation, as much as 40% of the season's sales happen ten days before Christmas (https://www.wsj.com/articles/holiday-shoppers-wait-until-the-last-minute-for-deals accessed on Dec 19, 2015). Academic researchers certainly recognize it and have investigated firms' optimal response to it from various directions; for example, Su [1], Aviv and Pazgal [2], Liu and Van Ryzin [3], Yin et al. [4], Lai et al. [5], Cachon and Swinney [6,7], to name a few.

Much of the existing literature on strategic customers focuses on a single product and assumes each customer purchases at most one unit of a product. Hence, the customer's decision is *whether* and *when* to make a purchase. However, experiences tell that customers may demand more than one unit for certain products, a phenomenon not covered in the literature. For example, when customers purchase wardrobe essentials such as formal apparel (ties and dressing shirts) and casual clothing (polos and tees), they may purchase multiple units and distribute usage evenly between them because each unit can wear out quickly if used too often. In another example, when customers buy bread, they may desire multiple units, in which case, they must decide how many to purchase in the morning or in the afternoon because fresh bread baked in early mornings are often sold at discounted prices in late afternoons. Similar examples include vegetables (particularly leafy greens) which are sold at clearance prices in early evenings. Notice first that customers often value additional units of the same product less than the first unit, implying a diminishing margin for additional units. Second, the prices of some fashion apparel (e.g., newly designed tees) may vary significantly throughout the selling season; specifically, it is common practice to sell each item at a full price when the selling season begins and at a significantly lower price before the selling season closes. Facing this price adjustment, customers demanding multiple units of a product exhibit more complicated purchasing behavior compared with as if they demanded at most one unit, as their purchasing decisions must span two dimensions, that is, *when* to make a purchase and *how many* to purchase each time. Because customers do not value the additional purchase equally to the first, the results of multi-unit purchases cannot simply replicate those of single-unit purchases already revealed in the literature. Two questions arise naturally: how should strategic customers make purchasing decisions when they demand more than one unit of a product? What are the implications of such strategic purchasing behavior on a firm's pricing decision?

* Corresponding author.
E-mail addresses: disjinc@nus.edu.sg (C. Jin), qianliu@ust.hk (Q. Liu), allenwu@ust.hk (C. Wu).

https://doi.org/10.1016/j.orl.2020.12.012

We show that two critical factors – the value of the second unit and the strategic level of customers – have significant impacts on the firm's optimal pricing decision. First, the firm's prices can decrease with the value of the second unit. This happens when it is optimal to induce high-value customers to make an additional purchase either in the *first* period or in the *second* period. In this case, the firm reduces prices sharply for both periods to induce the purchase of the second unit. Second, the first-period price can increase as customers behave more strategically, in contrast to the case of the single-unit selling where the first-period price always decreases with the strategic level of customers. For medium to low second-unit valuations, once the strategic level of customers reaches a critical threshold, the selling price in the first period becomes so low that the firm would rather not choose to target the second-unit sales in the *second* period. Consequently, the firm increases prices in both periods sharply to create a new segmentation of the market. Likewise, when the value of the second unit is medium to high, there is a critical threshold of strategic level at which the firm increases prices again, but this time, only to forgo the second-unit sales in the *first* period. In essence, these results are driven by the intricate interplay between the strategic purchasing behavior and multi-unit demands, creating switches between different segmentations of the market and contributing to a new insight not seen in single-unit selling.

The multi-unit selling also has a different effect on consumer surplus compared to the single-unit selling. First, consumer surplus may not always increase with the second-unit value. This follows from the earlier result that the optimal prices in each period do not change monotonically and continuously with the second-unit value. Second, consumer surplus may not always increase with the strategic level of customers. For certain values of the second unit, as customers become more strategic, the benefit of introducing second-unit purchases can be insufficient to compensate for a reduced profit margin in the first period. The firm responds by increasing prices in both periods to forgo second-unit sales in some period, and consumer surplus plummets as a result. This is again in contrast to the single-unit selling where customers always enjoy higher surplus from a higher strategic level.

Related literature. Strategic customer behavior has gained increasing interest in the operations and revenue management community in recent decades. A number of papers study its impact on firms' operational decisions in a variety of dimensions. For example, Su [1], Aviv and Pazgal [2], Levin et al. [8] focus on understanding its impact on firms' optimal dynamic pricing under a fixed inventory level. Su and Zhang [9], Cachon and Swinney [6,7], Parlakturk [10], Shum et al. [11] examine its impact on supply chain performance, quick response in the fast fashion industry, product variety and product cost reduction, respectively. Liu and Van Ryzin [12], Ovchinnikov and Milner [13], Wu et al. [14], Huang et al. [15] study its impact in the presence of customer learning. Levin et al. [16], Liu and Zhang [17] analyze its impact on competing firms. Various mechanisms have been proposed to discourage customers from waiting strategically, such as capacity rationing (Liu and Van Ryzin [3]), posterior price matching (Lai et al. [5]), in-store display formats (Yin et al. [4]) and binding reservation schemes (Osadchiy and Vulcano [18]). More recently, Aflaki et al. [19] allow customers to endogenize their decisions of becoming strategic by exerting costly efforts and demonstrate the effect of such behavior on a firm's profit, consumer surplus and social welfare. All these existing papers consider strategic customers restricted to single-unit purchase; that is, each customer demands at most one unit of a product. Our work clearly differs from them by considering a richer and more general setting that involves multi-unit demands. One exception is Elmaghraby et al. [20] that study multi-unit demands in a bidding market. However, Elmaghraby et al. [20] assume identical values for each individual unit in multi-unit demands, whereas we consider diminishing margins for additional units which constitutes a key factor of our main results.

2. Model set-up

We study a firm selling a single product with a fixed marginal cost that is normalized to zero. The selling season consists of two successive periods, labeled 1 and 2. The firm can charge different prices (for each unit) in each period -p_1 in the first period and p_2 in the second period — to maximize the total profit. The firm preannounces the prices p_1 and p_2 and commits credibly to them. At the beginning of period 1, a unit mass of customers arrives. A key feature that distinguishes our model from previous studies of strategic customers is that our customers may purchase multiple units of the product. To simplify analysis, we assume that each customer can purchase up to two units. Customers' valuations of the first unit are heterogeneous and follow a uniform distribution over [0,1]. The valuations of the second unit are discounted. For example, when customers purchase wardrobe basics such as jeans, they generally value an additional unit of jeans less than the first. Specifically, we assume that a customer with valuation v for the first unit values the second at βv, where $\beta \in [0, 1]$ is the valuation discounting factor for the second unit. Customers behave strategically; they consider prices in both periods when they make purchasing decisions. All customers discount the surplus gained in the second period by $\delta \in (0, 1)$. A higher δ implies that customers are more willing to postpone their purchase to obtain a mark-down price, and we thus refer to δ as the strategic level of customers.

3. Model analysis

3.1. Optimal price decision

Each customer must decide *when* to make a purchase and *how many* units to purchase in each period. We use the notation X_{ij} to denote the corresponding quantity of a customer purchasing i units in the first period and j units in the second period, where $i, j \in \{0, 1, 2\}$ and $i + j \leq 2$. Specifically, we use $U_{ij}(v)$ to denote the payoff of a customer who has a valuation v for the first unit and decides to purchase i units in the first period and j units in the second period. Given prices (p_1, p_2), there are six purchasing options available to each customer: (1) purchasing two units in the first period: $U_{20}(v) = (1+\beta)v - 2p_1$; (2) purchasing two units in the second period: $U_{02}(v) = \delta[(1+\beta)v - 2p_2]$; (3) purchasing one unit in the first period: $U_{10}(v) = v - p_1$; (4) purchasing one unit in the second period: $U_{01}(v) = \delta(v - p_2)$; (5) purchasing one unit in each of the two periods: $U_{11}(v) = v - p_1 + \delta(\beta v - p_2)$; and (6) no purchase: $U_{00}(v) = 0$. Based on these six options, we characterize the market segmentation and the associated demand in Lemma 1.

Lemma 1. *Under prices (p_1, p_2), the market segmentation is characterized in Table 1.*

The possibility of multi-unit purchases significantly complicates the market segmentation, allowing five segments to coexist for a wide range of prices (p_1, p_2). Let S_{ij} denote the customer segment that purchases i units in the first period and j units in the second period, and D_{ij} denote its mass. The segment S_{20} is made up of customers with extremely high valuations, $v \geq \frac{p_1 - \delta p_2}{\beta(1-\delta)}$, who purchase two units in the first period. Customers with valuations

Table 1
Market segmentation.

Demand	$p_2(1-\delta+\beta\delta) < \beta p_1$	$p_2(1-\delta+\beta\delta) \geq \beta p_1$
D_{20}	$\mathbb{P}\left\{v : v \geq \frac{p_1-\delta p_2}{\beta(1-\delta)}\right\}$	$\mathbb{P}\left\{v : v \geq \frac{p_1-\delta p_2}{\beta(1-\delta)}\right\}$
D_{11}	$\mathbb{P}\left\{v : \frac{p_1-\delta p_2}{1-\delta} \leq v \leq \frac{p_1-\delta p_2}{\beta(1-\delta)}\right\}$	$\mathbb{P}\left\{v : \frac{p_2}{\beta} \leq v \leq \frac{p_1-\delta p_2}{\beta(1-\delta)}\right\}$
D_{02}	$\mathbb{P}\left\{v : \frac{p_2}{\beta} \leq v \leq \frac{p_1-\delta p_2}{1-\delta}\right\}$	0
D_{10}	0	$\mathbb{P}\left\{v : \frac{p_1-\delta p_2}{1-\delta} \leq v \leq \frac{p_2}{\beta}\right\}$
D_{01}	$\mathbb{P}\left\{v : p_2 \leq v \leq \frac{p_2}{\beta}\right\}$	$\mathbb{P}\left\{v : p_2 \leq v \leq \frac{p_1-\delta p_2}{1-\delta}\right\}$
D_{00}	$\mathbb{P}\{v : v \leq p_2\}$	$\mathbb{P}\{v : v \leq p_2\}$

Table 2
Optimal prices and revenue under price commitment.

β	p_1^*	p_2^*	R
$0 \leq \beta < \underline{\beta}$	$\frac{2}{\delta+3}$	$\frac{\delta+1}{\delta+3}$	$\frac{1}{\delta+3}$
$\underline{\beta} < \beta < \bar{\beta}$	$\frac{\beta(\delta+3)+2(1-\delta)}{\beta(\delta+3)+4}$	$\frac{\beta(\delta+3)}{\beta(\delta+3)+4}$	$\frac{\beta(\delta+3)+1-\delta}{\beta(\delta+3)+4}$
$\bar{\beta} < \beta \leq 1$	$\frac{4\beta}{(\beta+1)(\delta+3)}$	$\frac{2\beta(1+\delta)}{(\beta+1)(\delta+3)}$	$\frac{4\beta}{(\beta+1)(\delta+3)}$

that are intermediately high, $\max\left\{\frac{p_1-\delta p_2}{1-\delta}, \frac{p_2}{\beta}\right\} \leq v < \frac{p_1-\delta p_2}{\beta(1-\delta)}$, constitute the segment S_{11}. These customers also purchase two units, but their purchases are split across two periods. Their second unit has a moderate value that is insufficient to justify an immediate purchase so its purchase is better delegated to the second period. The purchasing decisions of customers with intermediate valuations, $\min\left\{\frac{p_1-\delta p_2}{1-\delta}, \frac{p_2}{\beta}\right\} \leq v < \max\left\{\frac{p_1-\delta p_2}{1-\delta}, \frac{p_2}{\beta}\right\}$, are more involved, as they constitute either the segment S_{10} or the segment S_{02} exclusively. If the second-period price adjusted by time discounting $\left(\frac{1-\delta+\beta\delta}{\beta}\right) p_2$ is not as attractive as the first-period price p_1, all these customers purchase one unit in the first period; otherwise, they wait till the second period and purchase two units. The purchasing decisions of these customers are unanimous, making one of the segments S_{10} and S_{02} empty. Followed is the segment S_{01} made up of customers with lower valuations, $p_2 \leq v < \min\left\{\frac{p_1-\delta p_2}{1-\delta}, \frac{p_2}{\beta}\right\}$, who can only afford to purchase their first unit in the second period. The rest whose valuations are below p_2 do not contribute to any demand.

We solve the firm's revenue optimization problem and characterize the optimal prices (p_1^*, p_2^*) in the next result .

Proposition 1. *Let* $\underline{\beta} \triangleq \frac{(\delta+1)^2}{(\delta+2)(\delta+3)}$ *and* $\bar{\beta} \triangleq \frac{\sqrt{\delta^2+6\delta+13}-2}{\delta+3}$. *The firm's optimal prices* (p_1^*, p_2^*) *and the resulting revenue are summarized in Table 2.*

The two critical thresholds of β ($\underline{\beta}$ and $\bar{\beta}$) highlight how the firm can exploit multi-unit sales to optimize market segmentation and maximize revenue. Recall that customers' purchasing decisions vary in two dimensions: a *quantity decision* that determines the number of units to purchase and a *timing decision* that allocates multi-unit purchases between periods. Correspondingly, the firm's pricing decisions also vary in two dimensions: when to stimulate the second-unit purchase and how to induce it in an early period. These critical decisions drive the two thresholds of β. When β is sufficiently small, the second unit has a marginal value that is too low to justify any purchase of it in either period. As β increases to hit the first threshold $\underline{\beta}$ (the thick dotted curve in Fig. 1), the second unit gains enough appeal to start to affect customers' quantity decisions. The firm responds by lowering the second-period price to stimulate the second-unit sales. The first-period price is also adjusted downward to stabilize the demand for the first unit. To explain the second threshold, note that the firm always has an incentive to steer customers, if they qualify for a second-unit purchase, to purchase it early rather than later at a higher price. However, an early purchase of the second unit is only possible when β increases to the second threshold $\bar{\beta}$ (the thick solid curve in Fig. 1) as it requires the second unit to have a sufficient appeal to high-value customers.

We next identify the comparative statics of optimal prices. In particular, we examine how the optimal prices change with the two critical factors, β and δ, respectively. We present a graphical illustration of Proposition 2 in Fig. 2.

Proposition 2.

(i) Both p_1^* *and* p_2^* *are non-monotonic in* β. *Specifically, both are constants for* $0 \leq \beta < \underline{\beta}$ *and increase piecewise in* β *for* $\underline{\beta} < \beta < \bar{\beta}$ *and* $\bar{\beta} < \beta \leq 1$, *and have a downward jump at* $\underline{\beta}$ *and* $\bar{\beta}$.

(ii) a. The first-period price p_1^* *is monotonic decreasing in* δ *when* $\beta \in \left[0, \frac{1}{6}\right] \cup \left[\frac{1}{3}, \frac{\sqrt{13}-2}{3}\right] \cup \left[\frac{\sqrt{5}-1}{2}, 1\right]$, *but is non-monotonic in* δ *otherwise. In the latter case,* p_1^* *decreases piecewise in* δ *for* $0 < \delta < \bar{\delta}$ *and* $\bar{\delta} < \delta < 1$, *and has an upward jump at* $\bar{\delta}$, *where*

$$\bar{\delta} \triangleq \begin{cases} \frac{5\beta-2+\sqrt{\beta^2+8\beta}}{2(1-\beta)}, & \text{if } \frac{1}{6} \leq \beta < \frac{1}{3}, \\ \frac{3\beta^2+4\beta-3}{1-\beta^2}, & \text{if } \frac{\sqrt{13}-2}{3} \leq \beta < \frac{\sqrt{5}-1}{2}. \end{cases} \tag{1}$$

b. The second-period price p_2^* *is always increasing in* δ.

Part (i) of Proposition 2 shows the impact of the second-unit value on the optimal prices. If β is much too small, $\beta < \underline{\beta}$, the second unit has a marginal value that is too low to be relevant and thus the optimal prices are independent of β. As β increases to $\underline{\beta}$, the firm cuts prices in both periods to sell the second unit to high-value customers, but only to sell it in the second period. Both prices then increase steadily in β until it reaches the second threshold $\bar{\beta}$, at which the firm cuts prices again to induce an early purchase of the second unit, causing the second jump in price. Both prices then increase continuously afterwards in response to the increased value of the second unit. Part (ii) of Proposition 2 shows the impact of the strategic level δ on the firm's optimal prices. When $\beta \in \left[0, \frac{1}{6}\right)$, the second unit has such a marginal value that no one purchases it. Thus, the optimal price in the first (second) period decreases (increases) in δ as in the single-unit selling. When $\beta \in \left(\frac{1}{6}, \frac{1}{3}\right)$, some high-valuation customers purchase the second unit in the second period. An increased strategic level decreases the first-period price; when δ reaches $\bar{\delta}$, the first-period price has dropped so far that the firm would rather not target the second-unit sales in the *second* period. In response, the firm raises prices in both periods so that the second unit becomes irrelevant again, leading the optimal price in the first (second) period to decrease (increase) in δ. When $\beta \in \left(\frac{1}{3}, \frac{\sqrt{13}-2}{3}\right)$, the second unit has an intermediate value that always supports a purchase in the second period irrespective of the strategic level so prices in both periods are monotonic in δ. When $\beta \in \left(\frac{\sqrt{13}-2}{3}, \frac{\sqrt{5}-1}{2}\right)$, customers with high valuations start to purchase their second unit in the first period when the strategic level is below $\bar{\delta}$, above which customers wait to purchase the second unit later. The firm then adjusts prices upward to abandon the second-unit sales in the *first* period. When $\beta \in \left(\frac{\sqrt{5}-1}{2}, 1\right]$, customers with high valuations always purchase their second unit in the first period regardless of the strategic level, so the optimal prices are again monotonic in δ.

3.2. Consumer surplus and social welfare

Having characterized the optimal prices in both periods, we now discuss the implications of multi-unit purchase on the firm's

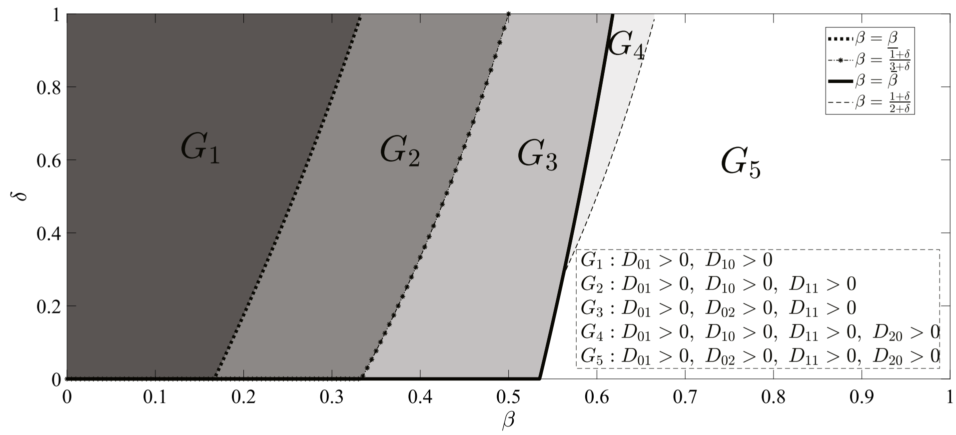

Fig. 1. Market Segmentation Under Optimal Prices.

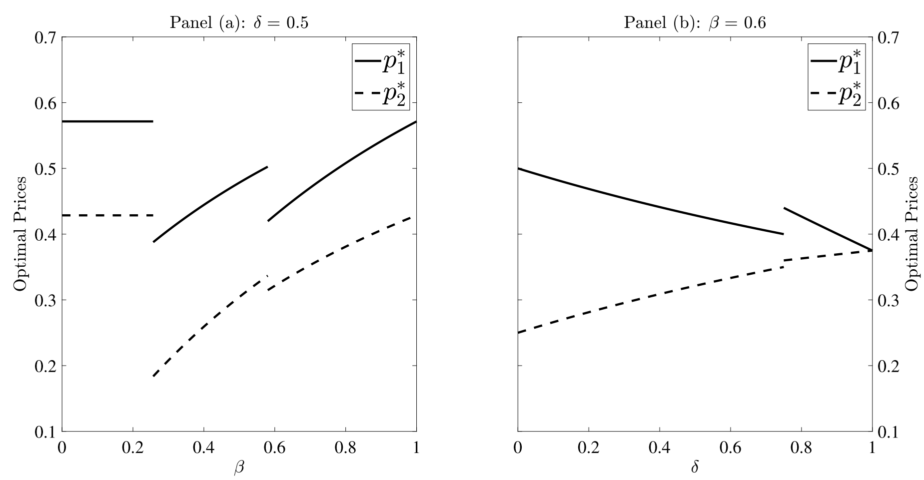

Fig. 2. Impact of β and δ on Optimal Prices.

revenue, customers and the society. The consumer surplus, denoted by CS, can be computed as follows:

$$CS = \int_0^1 \max\{U_{20}(v), U_{02}(v), U_{10}(v), U_{01}(v), U_{11}(v), 0\}\, dv,$$

and the social welfare $SW = CS + R$.

Proposition 3.

(i) The optimal revenue R is increasing in β and decreasing in δ.

(ii) a. The consumer surplus CS is non-monotonic in β. Specifically, CS is a constant for $0 \le \beta < \underline{\beta}$, piecewise decreasing in β for $\underline{\beta} < \beta < \bar{\beta}$ and $\bar{\beta} < \beta < \frac{2\sqrt{\delta^2+3\delta+8}}{\delta+3} - 1$, increasing in β for $\frac{2\sqrt{\delta^2+3\delta+8}}{\delta+3} - 1 \le \beta \le 1$ and has an upward jump at $\underline{\beta}$ and $\bar{\beta}$. The optimal consumer surplus is attained at $\beta = \underline{\beta}$ or $\beta = \bar{\beta}$.

b. The consumer surplus CS increases monotonically in δ when $\beta \in \left[0, \frac{1}{6}\right] \cup \left[\frac{1}{3}, \frac{\sqrt{13}-2}{3}\right] \cup \left[\frac{\sqrt{5}-1}{2}, 1\right]$ and non-monotonically in δ otherwise. In the latter case, CS increases piecewise in δ for $0 < \delta < \bar{\delta}$ and $\bar{\delta} < \delta < 1$ and has a downward jump at $\bar{\delta}$, where $\bar{\delta}$ is defined in (1). The optimal consumer surplus is attained at $\delta = \bar{\delta}$ or $\delta = 1$.

(iii) The social welfare SW increases in β. The impact of δ on the social welfare SW is non-monotonic and is shown in Table 3.

Fig. 3 gives a graphical illustration of Proposition 3. The firm can always take advantage of an increased second-unit value by adjusting prices to stimulate and manage second-unit sales. The social welfare also increases in β, so the socially optimum outcome is attained at $\beta = 1$. The consumer surplus, however, is non-monotonic in β and has jumps at critical thresholds where the optimal segmentation switches. For a marginal value of β, no customers ever purchase the second unit, and thus the consumer surplus is irrelevant to β until it hits the first threshold $\underline{\beta}$. At this threshold, the prices in each period take a downward jump in order to induce second-unit purchases, resulting in an upward jump for the consumer surplus. The consumer surplus then decreases in β afterwards because the firm increases prices in a way that reaps most of the benefits of the increased second-unit value. As β hits another threshold $\bar{\beta}$, the consumer surplus takes another upward jump because the firm cuts prices in both periods again to induce an early purchase of the second unit. As β grows further above $\bar{\beta}$, two interacting effects jointly render the consumer surplus non-monotone. An increased β implies a higher value obtained from a second-unit purchase, and meanwhile it enhances the firm's potential to extract surplus through pricing. For β slightly above $\bar{\beta}$, the latter effect dominates and the consumer surplus decreases. As β becomes sufficiently large, the second unit has a value sufficiently close to the first unit that the firm cannot extract surplus to a large extent. As a result, the consumer surplus increases for β close to 1. Although both the firm and society desire a higher value of the second unit,

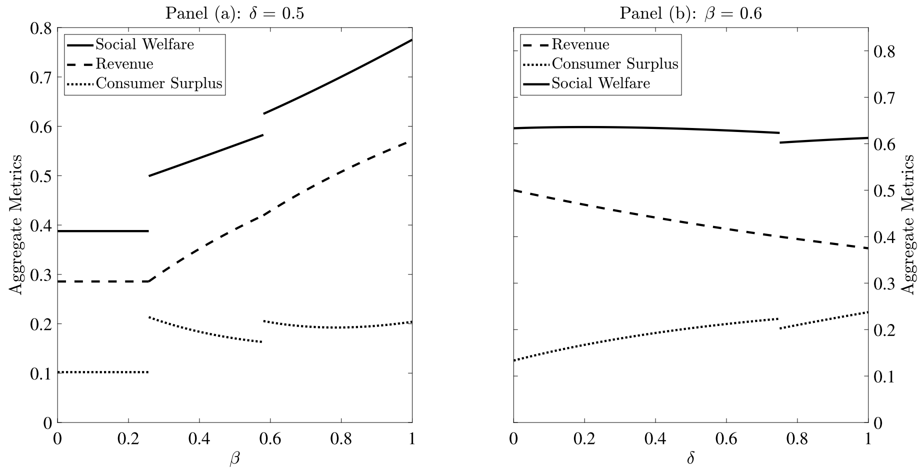

Fig. 3. Impact of β and δ on Firm's Revenue, Consumer Surplus and Social Welfare.

Table 3
Social welfare.

β	δ		
$[0, 1/6)$	$[0, 1/5): SW \nearrow$	$[1/5, 1]: SW \searrow$	
$[1/6, 9/44)$	$[0, \bar{\delta}): SW \nearrow$	$(\bar{\delta}, 1/5): SW \nearrow$	$[1/5, 1]: SW \searrow$
$[9/44, 1/3)$	$[0, \bar{\delta}): SW \nearrow$	$(\bar{\delta}, 1]: SW \searrow$	
$[1/3, (\sqrt{13}-2)/3)$		$SW \nearrow$	
$[(\sqrt{13}-2)/3, (\sqrt{89}-5)/8)$	$[0, \bar{\delta}): SW \nearrow$	$(\bar{\delta}, 1]: SW \searrow$	
$[(\sqrt{89}-5)/8, (\sqrt{5}-1)/2)$	$[0, 1/5): SW \nearrow$	$[1/5, \bar{\delta}): SW \searrow$	$(\bar{\delta}, 1]: SW \nearrow$
$[(\sqrt{5}-1)/2, 1]$	$[0, 1/5): SW \nearrow$	$[1/5, 1]: SW \searrow$	

customers benefit most from an intermediate second-unit value. Proposition 3 also shows the impact of customers' strategic level on the firm's profit, consumer surplus and social welfare. Unlike the single-unit selling, customers in the multi-unit setting may not always benefit from being highly strategic. For certain values of β, at the critical threshold of δ, the firm increases prices, leading to fewer purchases and a downward jump in consumer surplus. Therefore, *both the firm and customers may suffer from a high strategic level at certain point.* Notice that the consumer surplus is maximized at a strategic level of either $\bar{\delta}$ or 1. The impact of the strategic level on social welfare, however, is very elusive and is highly dependent on the value of β, with detailed results shown in Table 3.

4. Dynamic pricing

In this section we briefly discuss the dynamic pricing problem of a firm selling multiple units to strategic customers. Dynamic pricing is relevant in settings where commitment devices (business regulations, reputation considerations, etc.) are lacking. Under dynamic pricing, the firm announces the price for each period at the beginning of that period and customers make purchase decisions based on their beliefs on future prices. We follow the convention (e.g., [9] and [6]) to study *rational equilibrium* in which customers' beliefs on the second-period price are consistent with the actual price set by the firm to maximize the profit-to-go. Previous studies (e.g., [19]) have considered this problem in the single-unit setting corresponding to $\beta = 0$ and demonstrated that any first-period price $p_1 \in (0, 1)$ can be sustained in a rational equilibrium by inducing a second-period price $p_2 = p_1/(2-\delta)$. In our multi-unit setting, we find that the condition becomes more stringent for the first-period price p_1 to be sustained in equilibrium. Specifically, when $0 < \beta < 1/3$, there exists a sizable region $\left(\frac{(\beta+\sqrt{\beta+\beta^2})[2(1-\delta)+\beta(2-\delta)]+\beta\delta}{2(1+\beta)}, (1-\delta/2)(\sqrt{\beta+\beta^2}+\beta)\right)$ such that any first-period price that falls into this region cannot be sustained in a rational equilibrium. However, when $\beta = 1/3$, as long as $p_1 \geq 1-\delta/2$, there are two second-period prices sustained in equilibrium, $p_2 = 1/4$ or $1/2$.

We solve the firm's revenue optimization problem over all rational equilibria. We find that the new segmentation, although driven by a different pricing mechanism, is largely similar to that under price commitment. Specifically, we observe that the second-unit purchase in the second period starts with intermediate values of β. However, there is subtle difference as β further grows to one. The second unit is purchased in the first period under dynamic pricing unless the strategic level is not too high. But it is always purchased under price commitment regardless of the strategic level when β is close to one. This points to an inefficiency of dynamic pricing in inducing multi-unit purchases in an early period. Previous studies (e.g., [9]) have identified the relative strength of price commitment over dynamic pricing in the single-unit setting, and the same result continues to hold in our multi-unit setting as dynamic pricing is found to be less efficient in segmenting the market.

5. Discussions and conclusions

We examine several extensions to the base model (of price commitment) and summarize our main findings. (1) We examine the revenue loss when the seller ignores the behavior multi-unit purchases. If this was the case, the seller would set the prices as if $\beta = 0$. We numerically find that the revenue loss is the most significant for intermediate values of β and that this loss can be over 20% in some instances. (2) We consider a heterogeneous market of customers that differ in their second-unit values β. We particularly consider a market with half of its customers valuing the second unit (i.e., $\beta > 0$) and the other half not valuing the second unit (i.e., $\beta = 0$). We numerically find that the optimal prices in each period continue to be non-monotone with respect to β and δ, echoing with the main findings of the base model. (3) When customers purchase wardrobe essentials or bread, they may desire more than two units. We thus consider a setting in which customers purchase up to three units of a product. Noting the analytical difficulty resulting from more purchase options of each customer (ten options in total), we numerically compute the optimal prices in each period and find that our main results on the non-monotonicity of optimal prices with respect to β and δ continue to hold.

In summary, this paper studies a firm's optimal pricing decision when selling multiple units to strategic customers, which has rarely been examined in the literature on strategic customers thus far. This research is relevant for certain type of products such as wardrobe basics, for example, classical dressing shirts and pants, where customers often demand more than one unit of a product and need to allocate their purchasing quantities across different time periods. Compared with the single-unit setting, this problem is challenging because customers' decisions span two dimensions regarding when to make a purchase and how many to purchase each time, creating an involved market segmentation that significantly complicates the firm's pricing strategy. We find that the firm's optimal price in each period is a piecewise-defined function that depends primarily on two key factors: the value of the second-unit and the strategic level of customers. We show that the firm's optimal price in each period does not always increase monotonically in the second-unit value and that the firm may charge a higher first-period price as customers become more strategic. Consequently, customers may not always benefit from a higher second-unit value or strategic level. We also analyze the firm's dynamic pricing problem and explore several extensions to the base model, and find that our main results remain to hold in all those extensions.

Acknowledgments

The authors are supported by Singapore Ministry of Education Academic Research Fund [Tier 1, Grant R-253-000-144-133] and the Hong Kong Research Grants Council [Grants 16207119 and 16206618].

References

[1] X. Su, Intertemporal pricing with strategic customer behavior, Manage. Sci. 53 (5) (2007) 726–741.
[2] Y. Aviv, A. Pazgal, Optimal pricing of seasonal products in the presence of forward-looking consumers, Manuf. Serv. Oper. Manage. 10 (3) (2008) 339–359.
[3] Q. Liu, G.J. Van Ryzin, Strategic capacity rationing to induce early purchases, Manage. Sci. 54 (6) (2008) 1115–1131.
[4] R. Yin, Y. Aviv, A. Pazgal, C.S. Tang, Optimal markdown pricing: Implications of inventory display formats in the presence of strategic customers, Manage. Sci. 55 (8) (2009) 1391–1408.
[5] G. Lai, L.G. Debo, K. Sycara, Buy now and match later: Impact of posterior price matching on profit with strategic consumers, Manuf. Serv. Oper. Manage. 12 (1) (2010) 33–55.
[6] G.P. Cachon, R. Swinney, Purchasing, pricing, and quick response in the presence of strategic consumers, Manage. Sci. 55 (3) (2009) 497–511.
[7] G.P. Cachon, R. Swinney, The value of fast fashion: Quick response, enhanced design, and strategic consumer behavior, Manage. Sci. 57 (4) (2011) 778–795.
[8] Y. Levin, J. McGill, M. Nediak, Optimal dynamic pricing of perishable items by a monopolist facing strategic consumers, Prod. Oper. Manage. 19 (1) (2010) 40–60.
[9] X. Su, F. Zhang, Strategic customer behavior, commitment, and supply chain performance, Manage. Sci. 54 (10) (2008) 1759–1773.
[10] A.K. Parlakturk, The value of product variety when selling to strategic consumers, Manuf. Serv. Oper. Manage. 14 (3) (2012) 371–385.
[11] S. Shum, S. Tong, T. Xiao, On the impact of uncertain cost reduction when selling to strategic customers, Manage. Sci. 63 (3) (2017) 843–860.
[12] Q. Liu, G.J. Van Ryzin, Strategic capacity rationing when customers learn, Manuf. Serv. Oper. Manage. 13 (1) (2011) 89–107.
[13] A. Ovchinnikov, J.M. Milner, Revenue management with end-of-period discounts in the presence of customer learning, Prod. Oper. Manage. 21 (1) (2012) 69–84.
[14] S. Wu, Q. Liu, R. Zhang, The reference effects on a retailer's dynamic pricing and inventory strategies with strategic consumers, Oper. Res. 63 (6) (2015) 1320–1335.
[15] T. Huang, C. Liang, J. Wang, The value of "Bespoke": Demand learning, preference learning, and customer behavior, Manage. Sci. 64 (7) (2018) 3129–3145.
[16] Y. Levin, J. McGill, M. Nediak, Dynamic pricing in the presence of strategic consumers and oligopolistic competition, Manage. Sci. 55 (1) (2009) 32–46.
[17] Q. Liu, D. Zhang, Dynamic pricing competition with strategic customers under vertical product differentiation, Manage. Sci. 59 (1) (2013) 84–101.
[18] N. Osadchiy, G. Vulcano, Selling with binding reservations in the presence of strategic consumers, Manage. Sci. 56 (12) (2010) 2173–2190.
[19] A. Aflaki, P. Feldman, R. Swinney, Becoming strategic: Endogenous consumer time preferences and multiperiod pricing, Oper. Res. 68 (4) (2020) 1116–1131.
[20] W. Elmaghraby, A. Gulcu, P. Keskinocak, Optimal markdown mechanisms in the presence of rational customers with multiunit demands, Manuf. Serv. Oper. Manage. 10 (1) (2008) 126–148.