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Piracy and Bundling of Information Goods

Chen Jin [a], Chenguang (Allen) Wu [b], and Atanu Lahiri [c]

[a]Assistant Professor of Information Systems and Analytics, School of Computing, National University of Singapore, Singapore, 117417; [b]Assistant Professor of Industrial Engineering and Decision Analytcs, Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong; [c]Associate Professor of Information Systems, Jindal School of Management, University of Texas at Dallas, Richardson, TX 75080

ABSTRACT

Bundling is considered to be an effective pricing strategy for zero-marginal-cost information goods. Yet, in many information-goods markets, the effectiveness of bundling remains hard to ascertain. This is because information goods exhibit other characteristics as well, which can potentially interfere with a manufacturer's bundling decision. For instance, they are also prone to piracy, and it is not obvious what impact, if any, piracy can have on the efficacy of bundling. To address this issue rigorously, we reexamine the classic bundling problem with the backdrop of piracy and show that piracy can severely diminish the appeal of bundling to a monopolist seller. Evidently, bundling abets piracy and, in certain situations, so much so that the losses from piracy more than nullify the traditional benefits of bundling. This insight is in fact fairly generalizable. The implication for manufacturers of digital goods is that they need to take piracy into consideration in their bundling decision. In particular, they should consider refraining from bundling when the illegal products are close substitutes for the legal ones.

KEYWORDS
Bundling; information goods; mixed bundling; piracy; pricing

Introduction

Bundling is naturally appealing to producers of zero-marginal-cost information goods [4]. However, information goods are also piracy-prone, and it is not clear if bundling is actually profitable in the presence of piracy. Could piracy enhance or diminish the appeal of bundling? If it does, why and under what circumstances will it do so? And what are the ensuing implications?

These questions are of practical significance. For example, Feldman [10] blames the bundling of digital content for increased piracy and grumbles:

> "*You know what's free? Illegally downloaded movies. Piracy is back. For years, consumers griped about cable bundling—having to pay high prices for hundreds of channels that they never watched in order to get the handful they did watch. The unrealized dream was that at some point cable companies would relent and offer à la carte pricing, in which customers only paid for the channels they wanted. It appears now that the streaming market saturation has led to a refracted version of this problem, show bundling. Fans don't want all of a streaming service—they only want certain shows on it.*"

Feldman goes on to add:

CONTACT Atanu Lahiri atanu.lahiri@utdallas.edu Phone: +1 972.883.5934

> "*So, you could pay for a dozen different services to try and consume every new series and album and movie you're interested in legally . . . Or you could just pirate it.*"

Along similar lines, it has been observed that, to control piracy of its products, Microsoft sells standalone programs such as Word at a lower price than what it charges for the Microsoft Office bundle [25].

To address our research questions, we revisit the problem of bundling two zero-marginal-cost goods. We start with the case in which there is no piracy. In this benchmark setting, bundling predictably outperforms separate selling. The story changes quickly, though, when we incorporate piracy into the setup. We model the pirated version of a product as an imperfect substitute for the original. Moreover, piracy is costly to our consumers. The resulting model, which incorporates both bundling and piracy, is novel to the best of our knowledge, and it also leads us to new insights.

First, from a distance, it might appear that piracy presents a cheaper option to consumers whether a manufacturer uses bundling or not and, therefore, piracy should have no salient impact on the relative appeal of bundling vis-à-vis separate selling. This might lead one to conclude that the price-discrimination benefits of bundling should continue to prevail even when piracy is present. However, as we discover, such a conclusion will actually be premature, and there is in fact another countervailing force in play, the flexibility to consume à la carte—when consumers are interested in only one of the products, they can avoid paying up for the bundle and simply pirate the product they are interested in. This way, bundling can, and does increase the incentive to pirate, and consumers who do not pirate otherwise could do so in the presence of bundling. The implication is clear. The prevalence of piracy in certain information-goods markets is indeed attributable to bundling exactly as claimed by some in the popular press [10].

Second, even though bundling makes piracy more attractive, the converse is certainly not true. Piracy, in fact, makes bundling less attractive. We identify two distinct possibilities in equilibrium, one in which the gains from bundling are strong enough to outweigh losses from increased piracy, and another in which exactly the opposite is true. In general, when piracy is quite potent, that is, when the pirated product is a close substitute for its legal counterpart, bundling is unlikely to be the dominant strategy. This finding has important implications for information-goods manufacturers considering bundling as a pricing strategy.

We also examine the case where consumers' valuations for the bundle constituents are negatively correlated. In general, a strong negative correlation should increase the appeal of bundling, making it highly preferable from the seller's perspective. Yet, the findings mentioned above remain applicable even after a strong negative correlation is incorporated into the model. The insights obtained from our analysis of mixed bundling are also instructive. Mixed bundling, in general, is superior to pure bundling, and a manufacturer might therefore expect mixed bundling to outperform separate selling. However, as we explain, even this anticipation does not necessarily materialize in the presence of piracy.

We also consider that a fraction of the consumer market could behave ethically, refraining from illegal consumption. Even in this setting, the findings qualitatively extend, just as they do to the ones where the manufacturer is tasked with bundling more than two products or where the piracy cost is endogenous. All these analyses collectively indicate that a manufacturer of information goods needs to reassess the efficacy of bundling, particularly in markets where piracy is a serious concern.

Literature Review

The problem of bundling has been extensively studied [1, 4]. The main finding in this literature is that bundling decreases valuation heterogeneity among consumers, which translates to a greater pricing power for the manufacturer. In fact, the heterogeneity disappears completely when the number of products becomes large [4]. So, if there is no marginal cost, it becomes profitable for the manufacturer to sell just one bundle comprising all products.

Recently, researchers have studied different forms of bundling and their implications for information-goods markets [e.g., 2, 18]. Among works of particular interest, Prasad et al. [20] explored whether the usual efficacy of bundling carries over to a product market where network effects are significant. Interestingly, it does not, and separate selling can indeed become superior to bundling from a profit perspective. Likewise, Geng et al. [12] found that bundling could be inferior if consumers' average valuation per bundle component were to decrease with the bundle size, and Wu et al. [24] showed that the same could happen when consumers have valuation uncertainty about some of the components. We extend this line of research by demonstrating that bundling can be suboptimal in the presence of piracy.

Moving on to the literature on piracy, its primary focus has been on how a manufacturer might respond to piracy by using, among other things, preventive, and deterrent controls [14], nonlinear pricing [22], restrictive patching [3], product sampling [5], search-cost manipulation [11], strategic content delivery [15], and versioning [26]. The literature has also analyzed the welfare implications of piracy, especially the trade-off between private profits and public welfare [6, 9, 19].

Although the literature on piracy and bundling are each vast, their intersection remains largely unexplored. Gopal and Gupta [13] studied how a manufacturer can use bundling to combat a *sharing club*—a consortium of users promoting illegal sharing of digital products. Unlike Gopal and Gupta, we do not require that a consumer must either pirate all products or simply pirate none when a manufacturer employs bundling. This generalization has led us to a new finding: Even when the valuations and piracy costs are symmetric across products, bundling can actually be dominated by separate selling. More recently, researchers have considered the problem of bundling a software product with a cloud-based service [27, 28]. These papers are considerably different from ours. First, the cloud-based service has a non-zero marginal cost, which becomes a significant factor in the bundling decision. Second, the cloud-based service cannot be pirated; it is only the software product that is prone to piracy. An important finding in these papers is that a low marginal cost favors bundling [28, Observation 5], which echoes earlier research [4]. In contrast, despite considering zero-marginal-cost goods, we find that bundling can actually be suboptimal from the seller's perspective.

Since piracy can be thought of as a shadow competitor to the legal channel, our work is also related to the growing literature on competitive bundling. In a study that examines a horizontally-differentiated duopoly, Zhou [30] showed that if both firms were to bundle, they would both lose profit-wise (when compared to the case of both selling separately). In contrast, our manufacturer will gain from bundling even if the pirated products were available only as a bundle. A key reason for this difference is that we have a vertical setup in which the pirated goods are of lower quality compared to their legal counterparts and our shadow competitor does not make any pricing decisions. The shadow competitor does not control the decision to bundle either, so the pirated products might also be available à la carte. And, if so, separate selling will dominate bundling profit-wise only if the cost of piracy is moderate and the quality of pirated products is close to that of legal ones. Roels et al. [21] examined how two firms may respond to each other's decision to bundle in a competitive setting and showed that if, in a pure Bertrand duopoly, one firm were to bundle while the other sold separately, the latter will make zero profit, making such an outcome impossible in equilibrium. Such a result is not observed in our setting, and piracy does remain in force for the most part (except when the piracy cost is too high and piracy is trivially extinct). Evidently, our results are context-specific and cannot be inferred from the broader literature.

The rest of this paper is structured as follows. First, as a benchmark, we will discuss a setting without piracy. In this benchmark setting, consistent with prior literature, bundling does outperform separate selling. Next, we will incorporate piracy into our setup and recalculate the monopolist's optimal revenue under both separate selling and pure bundling; we will show that bundling may not be optimal when piracy is present. We will carry out a number of robustness checks. Finally, we will conclude with an analysis of mixed bundling.

Model Preliminaries

Consider a monopolist that produces and sells two information goods to a consumer market of size one. The marginal cost is zero for both products, and the development costs are sunk. Consumers' valuations for product $i \in \{1, 2\}$, denoted v_i , are uniformly distributed over $[0, 1]$. A consumer's valuations for the two products are mutually independent. Our notation and the related definitions are shown in Table 1, and a modeling analysis sequence flowchart in Figure 1 to orient the reader to this and the latter sections' analysis.

Benchmark Case of No Piracy

If the firm sells the two products separately, it will charge $p_1 = p_2 = \frac{1}{2}$ for each product and earn a total profit of 0.5 . If it sells the two products as a bundle at price p , a consumer will compare his $v_1 + v_2$ with p to make the purchase decision. The distribution for $v_1 + v_2$ is triangular and is more concentrated around the mean when compared to the uniform

Table 1. Modeling Notation and Definitions

Notation	Definition	Comments
v_i	Consumer valuation for product i	v_i is uniformly distributed over $[0, 1]$
p_i	Price of product i	Optimal value of p_i is denoted p_i^*
p	Bundle price	Optimal value of p is denoted p^*
δ	Relative quality of pirated product	$\delta \in (0, 1)$; $1 - \delta$ is the quality gap between legal and pirated versions of a product
r	Piracy cost (per product pirated)	Includes acquisition, search, legal costs, etc.
q_i	Demand for product i	—
q	Demand for the bundle	—
r_1	A threshold value for r , which depends on δ	If $r < r_1$, piracy exists under both bundling and separate selling
r_2	A threshold value for r , which depends on δ	If $r \geq r_2$, piracy ceases to exist under both bundling and separate selling

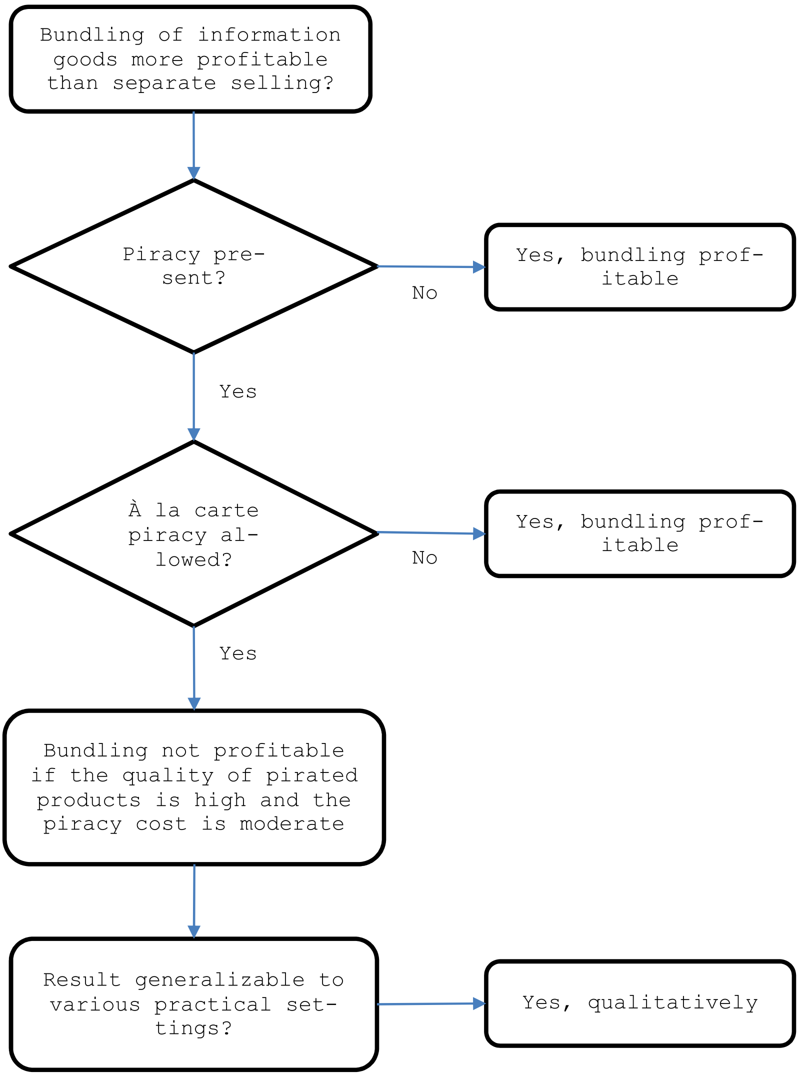

Figure 1. Modeling Analysis Sequence Flowchart

distribution. Accordingly, the optimal bundle price will become $\sqrt{\frac{2}{3}}$ and the corresponding profit, approximately 0.544 . Thus, in the absence of piracy, bundling is indeed more profitable than separate selling.

Model with Piracy

We borrow the standard piracy setup from prior literature and model the pirated version of a product as its imperfect substitute [e.g., 3, 16]:

- **Assumption 1 (Consumer's valuation for pirated product).** *A Consumer's valuation for the illegal version of product $i \in \{1,2\}$ is δv_i, where v_i is his valuation for the legal one; $\delta \in (0,1)$ represents the relative quality of the pirated version vis-à-vis the legal one.*

Consistent with prior research, we also assume that piracy is costly to consumers:

- **Assumption 2 (Expected cost of piracy).** *The expected piracy cost per product is $r \geq 0$.*

The parameter r subsumes all piracy-related costs, including the payment made to pirate suppliers and the search cost involved in locating the pirated product [e.g., 11]. Since a consumer will incur such costs for each act of piracy, the total cost to him should be proportional to the number of products pirated. Further, each instance of piracy is punishable under the law by a separate penalty [19], and thus, the expected legal cost too "*increases monotonically with . . . the number of products*" [13, p. 1949]. Now, although we assume r to be the same for all consumers, we have done numerical experiments to confirm that the main insights remain applicable even in the presence of some heterogeneity.

When the manufacturer sells separately, a consumer with valuation v_i buys product $i \in \{1,2\}$ if and only if his individual-rationality and incentive-compatibility constraints, $v_i - p_i \geq 0$ and $v_i - p_i \geq \delta v_i - r$, are satisfied. So, the legal demand for i is simply $q_i(p_i) = 1 - \max\{\frac{p_i - r}{1-\delta},\ p_i\}$. The manufacturer's problem is to maximize $\sum_{i=1}^{2} p_i q_i(p_i)$. This problem is separable in i , and as shown in prior research [16, 19], the optimal price p_i^* is as follows:

$$p_i^* = \begin{cases} \frac{1-\delta+r}{2}, & \text{if } r < \frac{\delta(1-\delta)}{2-\delta}, \\ \frac{r}{\delta}, & \text{if } \frac{\delta(1-\delta)}{2-\delta} \leq r \leq \frac{\delta}{2}, \\ \frac{1}{2}, & \text{otherwise.} \end{cases}$$

Essentially, when r is small, the monopolist tolerates piracy and sets a price of $\frac{1-\delta+r}{2}$. When r is moderate, the manufacturer is able to stave off piracy by holding p_i to the *limit price* of $\frac{r}{\delta}$. Finally, when r is high, it regains its full monopoly power and charges the benchmark price of $\frac{1}{2}$.

Bundling in the Presence of Piracy

We now turn to our main research question: What happens if the products are sold as a bundle in the presence of piracy? Since the expected surplus from pirating product i is $\delta v_i - r$ (see Assumption 2), given a bundle price p , a consumer buys the bundle if and only if $v_1 + v_2 - p \geq (\delta v_1 - r)^+ + (\delta v_2 - r)^+$. Accordingly, four consumer segments are possible:

- **Segment 1 (Consumers who do not consider piracy).** A consumer with $v_1 \leq \frac{r}{\delta}$ and $v_2 \leq \frac{r}{\delta}$ buys the bundle if and only if $v_1 + v_2 \geq p$.
- **Segment 2 (Consumers who might pirate either or both products).** A consumer $v_1 > \frac{r}{\delta}$ and $v_2 > \frac{r}{\delta}$ buys the bundle if and only if $v_1 + v_2 \geq \max\{p, \frac{p-2r}{1-\delta}\}$.

- **Segment 3 (Consumers who never pirate Product 1 but might pirate Product 2).** A consumer with $v_1 \le \frac{r}{\delta}$ and $v_2 > \frac{r}{\delta}$ buys the bundle if and only if $v_1 + (1-\delta)v_2 \ge p - r$.
- **Segment 4 (Consumers who never pirate Product 2 but might pirate Product 1).** A consumer with $v_1 > \frac{r}{\delta}$ and $v_2 \le \frac{r}{\delta}$ buys the bundle if and only if $(1-\delta)v_1 + v_2 \ge p - r$.

The total demand for the bundle, $q(p)$, is simply the sum of the demands from these four segments. As is apparent from the final expression for $q(p)$ provided in the Appendix, Equation (A1), ten different configurations can emerge depending on the values of p, r, and δ. In Figure 2, we illustrate one of those ten, specifically, what happens if $\frac{\delta(1-\delta)}{2-\delta} \le r < \delta$ as well as $\frac{2r}{\delta} < p \le 1 - \delta + r + \frac{r}{\delta}$; this is essentially the fourth sub-case under Case (ii) in (A1). To identify the four segments, we partition the consumer market—that is, the (v_1, v_2) space—using the lines $v_1 + (1-\delta)v_2 = p - r$, $(1-\delta)v_1 + v_2 = p - r$, and $v_1 + v_2 = \max\{p, \frac{p-2r}{1-\delta}\} = \frac{p-2r}{1-\delta}$. Note that $\frac{p-2r}{1-\delta} > p$ has to hold since $p > \frac{2r}{\delta}$ holds in this sub-case. Further, the conditions $p > \frac{2r}{\delta}$ and $r \ge \frac{\delta(1-\delta)}{2-\delta}$ together imply that $p > 1 - \delta + r$ also holds. This, in turn, ensures that the point $(1, p - (1 - \delta + r))$ —marked as X in Figure 2—is strictly above the v_1 -axis. Likewise, the point marked Y is located to the right of the v_2 -axis.

It follows from the definitions that the demand from Segment 3 ought to be the region trapped between $v_1 = \frac{r}{\delta}$ and $v_1 + (1-\delta)v_2 = p - r$ in Figure 2, that is, the one labeled S3. The demand from Segment 4 is similar; it is labeled S4. From the definition of Segment 2, it is also clear that the demand from this segment is the darker of the shaded regions, the one labeled S2. Finally, it is apparent from the figure that, in this particular sub-case, there can

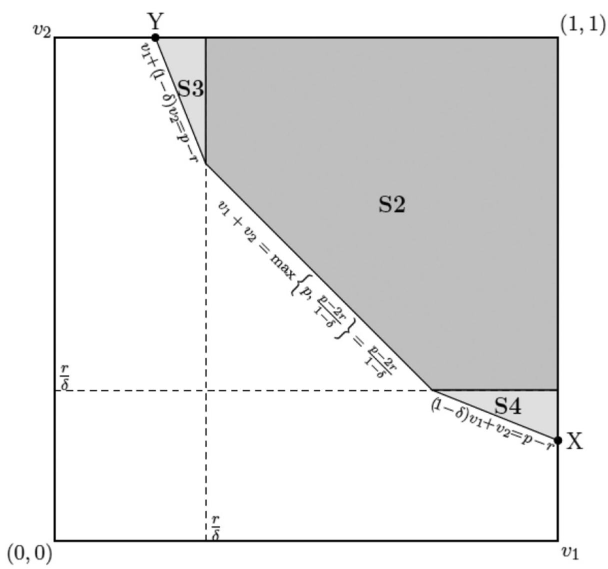

Figure 2. Consumer Segments Contributing to the Bundle Demand in the Presence of Piracy

be no demand from Segment 1. Accordingly, the total demand, $q(p)$, is simply the sum of the areas of the three shaded regions. The demands for the other nine sub-cases can be derived in a similar way, albeit a different picture is needed for each.

- **Proposition 1 (Optimal bundle price with piracy)**. *The optimal bundle price in the presence of piracy, p^*, is as shown in Table 2.*

The equilibrium demand can be obtained by substituting the optimal price in Table 2 into $(A1)$, and the optimal profit, by multiplying the resulting demand by the price. As expected, the optimal profit and price are both non-decreasing in r and non-increasing in δ . These trends are shown in Figure 3, which illustrates both the scenarios listed in the table: $\delta \leq \frac{2}{3}$ and $\delta > \frac{2}{3}$. The solid line in each panel shows how the per-product optimal price under bundling, $\frac{p^*}{2}$, changes with r . The dashed line represents the per-product optimal price under separate selling, p_i^* , discussed in the previous section. When r becomes large, both lines, solid and dashed, become flat and coincide with their respective monopoly prices in the benchmark case in which there is no piracy; to be specific, p_i^* becomes 0.5 , and $\frac{p^*}{2}$, approximately 0.408 .

The thresholds shown in the figure, $r_1 = \frac{\delta(1-\delta)}{2-\delta}$ and $r_2 = \delta\sqrt{\frac{2}{3}}$, represent the points beyond which the demand for the pirated product becomes zero under separate selling and bundling, respectively. A few observations are in order here. Until r_1 , both p_i^* and $\frac{p^*}{2}$ rise in a similar fashion; this is because, irrespective of the manufacturer's strategy, piracy is present in the market and it impacts both separate selling and bundling in a similar way. Recall from the previous section that once r_1 is crossed, however, the manufacturer switches to limit pricing, if selling separately; piracy disappears as a result, even though its threat remains and forces the manufacturer to take r into consideration in the pricing decision.

Table 2. Optimal Bundle Price in the Presence of Piracy

Range of δ	Optimal price, p^*	Condition on r
$\delta < \frac{2}{3}$	$\frac{4r+\sqrt{2\left(3\delta^2-6\delta+5r^2+3-\frac{3r^2}{\delta}\right)}}{3}$	if $r \leq \delta\sqrt{\frac{1-\delta}{6-\delta}}$
	$\frac{4r+\sqrt{2\left(3-\frac{3r^2}{\delta}+5r^2-3\delta^2\right)}}{3(1+\delta)}$	if $\delta\sqrt{\frac{1-\delta}{6-\delta}} < r < \delta\sqrt{\frac{2}{3}}$
	$\sqrt{\frac{2}{3}}$	if $r \geq \delta\sqrt{\frac{2}{3}}$
$\delta \geq \frac{2}{3}$	$\frac{4r+\sqrt{2\left(3\delta^2-6\delta+5r^2+3-\frac{3r^2}{\delta}\right)}}{3}$	if $r \leq \frac{\delta(1-\delta)}{3\delta-2}\left(\sqrt{4-\frac{2}{\delta}}-1\right)$
	$\frac{4\delta r+8\delta-4-4\delta^2+A}{3(2\delta-1)}$	if $\frac{\delta(1-\delta)}{3\delta-2}\left(\sqrt{4-\frac{2}{\delta}}-1\right) < r < \frac{\delta(2-\delta)}{6-\delta}$
	$\frac{4-\sqrt{2\left(3\delta-6r+2+\frac{3r^2}{\delta}\right)}}{3}$	if $\frac{\delta(2-\delta)}{6-\delta} \leq r < \frac{\delta}{3\delta-2}\left(3\delta-1-\sqrt{4-\frac{2}{\delta}}\right)$
	$\frac{4r+\sqrt{2\left(3-\frac{3r^2}{\delta}+5r^2-3\delta^2\right)}}{3(1+\delta)}$	if $\frac{\delta}{3\delta-2}\left(3\delta-1-\sqrt{4-\frac{2}{\delta}}\right) \leq r < \delta\sqrt{\frac{2}{3}}$
	$\sqrt{\frac{2}{3}}$	if $r \geq \delta\sqrt{\frac{2}{3}}$

Note: $A = \sqrt{2\left(\left(3\left(\sqrt{\delta}-\frac{1}{\sqrt{\delta}}\right)^2+2\delta^2\right)\left(r-\frac{\delta(2\delta+3)(1-\delta)^2}{2\delta^3+3\delta^2-6\delta+3}\right)^2+\frac{6(1-2\delta)^2(1-\delta)^2}{3(1-\delta)^2+2\delta^3}\right)}$

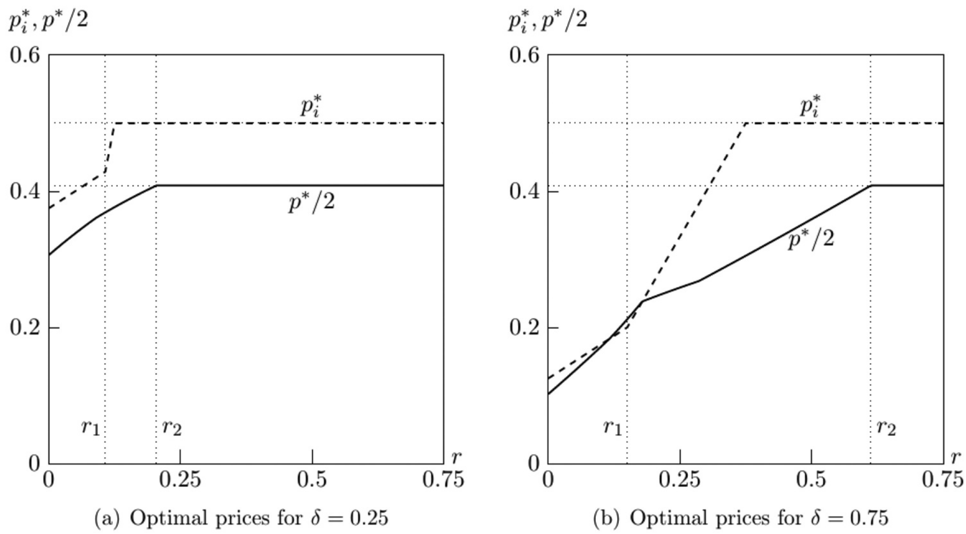

Figure 3. Optimal Per-Product Price under Bundling ($p^*/2$) vs. Separate Selling (p_i^*)

Also, recall that this limit price is $\frac{r}{\delta}$, which rises linearly with r . Piracy becomes completely irrelevant only after $\frac{r}{\delta}$ reaches the benchmark price of $\frac{1}{2}$, fully restoring the monopolist's pricing power.

Interestingly, no such limit pricing is possible under bundling, primarily because consumers' valuations for the two products are independent and, depending on his own valuations, a consumer can suitably decide to pirate either or both products as long as the cost to do so is not extremely high. This is precisely why, unlike $\frac{r}{\delta}$ which rises rapidly, $\frac{p^*}{2}$ rises rather gradually until r_2 , where it eventually attains its benchmark level. Naturally, the two price curves, p_i^* and $\frac{p^*}{2}$, start diverging between r_1 and r_2 . It is apparent from a quick comparison of the two panels in Figure 3—Panel (a) where $\delta = 0.25$ and Panel (b) where $\delta = 0.75$ —that this divergence can be quite significant at a large value of δ . The lesson is that, if δ is large and r is between r_1 and r_2 , the adverse effect of piracy is considerably more pronounced under bundling than under separate selling. So, we ought to ask: Can bundling still be the optimal strategy across the board? Specifically, can it retain its dominance when r is between r_1 and r_2 ?

Figure 4 depicts the net impact of piracy on the efficacy of bundling. It shows how the optimal profit resulting from Proposition 1 compares with that under separate selling. In the figure, the region above the $r = \delta$ line is uninteresting as piracy is trivially a non-issue there. In the region below, which is both interesting and non-trivial, selling separately dominates in the shaded portion and bundling, in the rest of the (δ, r) space. Further, as δ increases, so does the range of values of r for which selling separately is optimal. This is reflected in the funnel-like shape of the shaded region. The shape is instructive. Since, in many real-world contexts, pirated products are reasonable substitutes for legal ones—that is, δ is often close to unity—piracy ought to merit serious consideration from digital-goods manufacturers. Instead of presuming that bundling is ubiquitously optimal, each manufacturer should carefully evaluate its options.

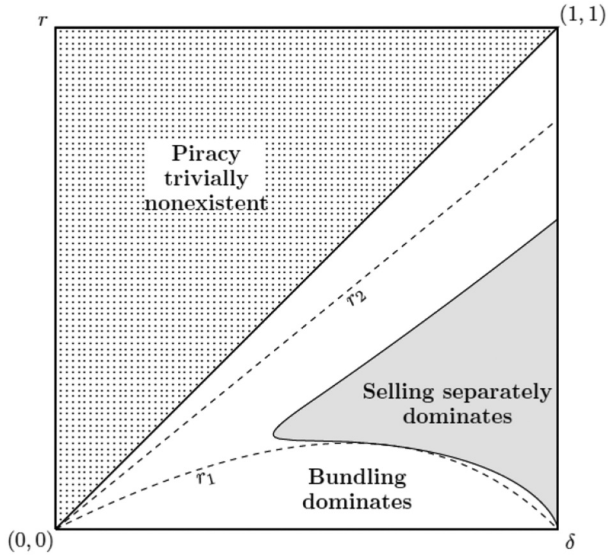

Figure 4. Optimality of Bundling in the Presence of Piracy

What explains such an impact of piracy on the optimality of bundling? To understand, we need to pay attention to the dashed lines in Figure 4, r_1 and r_2 . As discussed already, below r_1 , r is so low that piracy poses a considerable threat to the manufacturer regardless of its decision to bundle. In other words, it hurts bundling and separate selling alike, which is why the usual dominance of bundling remains intact. As r increases beyond r_1 , however, piracy ceases to exist under separate selling, but as long as r is not above r_2 , it persists under bundling. This is because a greater number of consumers now prefer pirating just one product to buying the bundle comprising both. Note that the cost of pirating both products, $2r$, is now too high, and at the same time, the bundle price, which is increasing in r , is high as well. All these factors, together, result in a higher incentive for consumers to pirate selectively. Thus, between r_1 and r_2 , piracy affects bundling disproportionately more—and especially so if δ is also large—causing the funnel-shaped region in Figure 4 to emerge. Finally, as r increases beyond r_2 , piracy fully subsides irrespective of the manufacturer's strategy, restoring the usual dominance of bundling. In summary, piracy can severely and adversely impact the efficacy of bundling when the piracy cost is moderate and the pirated products are reasonably good substitutes for the legal ones.

The broader takeaway is apparent. Piracy makes bundling less profitable when bundling exacerbates consumers' incentives to pirate. To the best of our knowledge, this mechanism is amiss in the literature, and prior theoretical works to look at this issue merely reaffirmed the superiority of bundling in the presence of piracy [13, Proposition 4]. We can replicate the results in prior research if we too place the restriction that consumers must pirate both bundle constituents or none when the manufacturer uses bundling:

- **Proposition 2 (Bundle price without à la carte pirating)**. *If consumers were not allowed the option to pirate à la carte, the optimal bundle price would be as follows, and bundling would always be more profitable.*

$$p^* = \begin{cases} \frac{4r+\sqrt{6(1-\delta)^2+4r^2}}{3}, & \text{if } r \le \frac{\delta}{2}\sqrt{\frac{2(1-\delta)}{3-\delta}}, \\ \frac{2r}{\delta}, & \text{if } \frac{\delta}{2}\sqrt{\frac{2(1-\delta)}{3-\delta}} < r \le \frac{\delta}{2}\sqrt{\frac{2}{3}}, \\ \sqrt{\frac{2}{3}}, & \text{if } r > \frac{\delta}{2}\sqrt{\frac{2}{3}}. \end{cases}$$

According to Proposition 2, if pirated products are available only as a bundle, the idea of a limit price becomes relevant once again; in fact, here, the limit price is $\frac{2r}{\delta}$, which is $\frac{r}{\delta}$ per product and thus exactly the same as that under separate selling. So, at moderate values of r , p^* described in Proposition 2 diverges from that in Proposition 1 similar to the way the dashed line in Figure 2 does from the solid one. This is precisely why bundling becomes dominant again.

Recall from the Introduction of this article that bundling has often been blamed for higher piracy rates in certain software and media markets because, unlike piracy, bundling denies consumers the option to consume selectively [10]. Clearly, Proposition 2 will not apply to such markets. Put another way, it is in such markets where we should expect Proposition 1 to hold true and piracy to have a significant adverse impact on the efficacy of bundling. A case in point is the software market where consumers looking to pirate a product are not required to install other products included in the same software suite. The context for TV show bundling is quite similar in this respect, as consumers interested in a popular show may not want to spend the time downloading and watching other shows included in a cable TV plan or subscription service. The desire to pirate selectively is clearly evident from the record-breaking piracy of the HBO series titled "The Game of Thrones" [16]. In the music market, too, a similar situation could occur, provided that illegal sharing sites offer consumers the option to download singles. Interestingly, there is empirical evidence to suggest that with the emergence of digital singles (e.g., sold through Apple's iTunes till 2019 and then replaced by Apple TV, Apple Music, and Podcasts), consumers' interest in piracy waned somewhat, as they no longer needed to purchase an entire album only to enjoy a fraction of it [17].

Nevertheless, consumer advocates, bloggers, and policymakers should be mindful when criticizing the pricing policies of digital goods manufacturers. This is because bundling price-discriminates against consumers, but it can also exacerbate piracy and thereby result in a higher surplus from illegal use. Thus, aggregate consumer welfare might be higher under bundling. To study this issue rigorously, we compare the aggregate consumer surplus under bundling with that under separate selling. Figure 5 shows the results.

As is evident from Figure 5, the manufacturer's incentives are surprisingly aligned with consumers under most circumstances. Specifically, in the darker part of the shaded region, both prefer separate selling, and in the transparent, unshaded region, both prefer bundling. Among the two lightly-shaded portions, in the bottom one, consumers prefer separate selling but the manufacturer does not, while the opposite holds in the top one. It is only in these portions that the manufacturer's preference for bundling differs from that of consumers. The lesson is clear: unqualified and unbridled criticism of bundling strategies may not be in the collective interest of consumers.

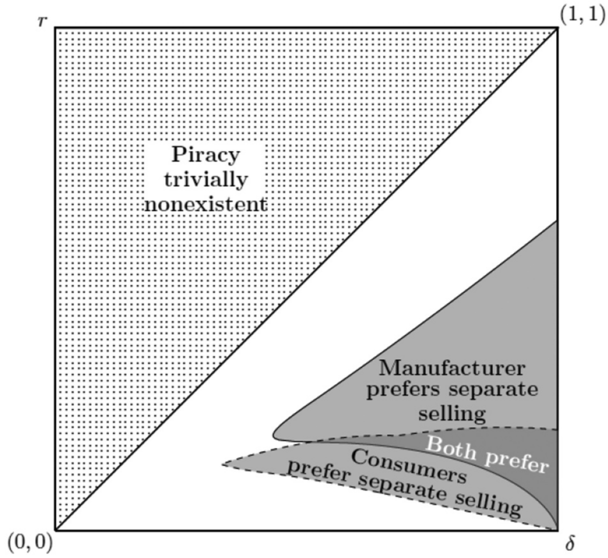

Figure 5. Manufacturer and Consumers' Preference for Separate Selling

Interdependent Product Valuations

Bundling is usually more profitable when consumer valuations for the bundle constituents are negatively correlated [1]. Are our results with respect to piracy then robust to the presence of a negative correlation? To answer, it is sufficient to examine the extreme case of a perfect negative correlation. In such a case, if a consumer's v_1 is v , his v_2 is simply $1 - v$. Since $v_1 + v_2 = 1$ for every consumer, in the absence of piracy, the manufacturer can extract all consumer surplus by simply selling a bundle for $p = 1$, which clearly outperforms any other conceivable pricing strategy. The profit from separate selling—irrespective of whether there is piracy—is not impacted by correlation, as it depends only on the marginal distributions of v_1 and v_2 . Moving on to the bundling problem in the presence of piracy, a consumer purchases the bundle if and only if $1 - p \geq (\delta v - r)^+ + (\delta(1 - v) - r)^+$. The following result is immediate.

- **Proposition 3 (Bundling when valuations have perfect negative correlation)**. *If consumer valuations for the two products exhibit a perfect negative correlation, the optimal bundle price will be as shown in Table 3.*

In Figure 6, we compare the optimal revenue obtained from Proposition 3 with that under the separate selling. The figure shows two shaded regions: The lighter one (inclusive of the darker region inside) is the same as that in Figure 4. The darker region is the one where bundling is suboptimal now, that is, in the presence of a perfect negative correlation. From the relative sizes of the two regions, it becomes apparent that a negative correlation indeed enhances the appeal of bundling. Nevertheless, there is still a substantial region where bundling remains suboptimal.

Table 3. Optimal Bundle Price with Piracy and Perfect Negative Correlation

Range of δ	Optimal price, p^*	Condition on r
$\delta < \frac{2}{3}$	$1 - \delta + r$	if $r < \delta$
	1	if $r \geq \delta$
$\delta \geq \frac{2}{3}$	$1 - \delta + 2r$	if $r < \frac{\delta}{2} - \frac{1}{3}$
	$\frac{1+r}{2} - \frac{\delta}{4}$	if $\frac{\delta}{2} - \frac{1}{3} \leq r < \frac{3\delta}{2} - 1$
	$1 - \delta + r$	if $\frac{3\delta}{2} - 1 \leq r < \delta$
	1	if $r \geq \delta$

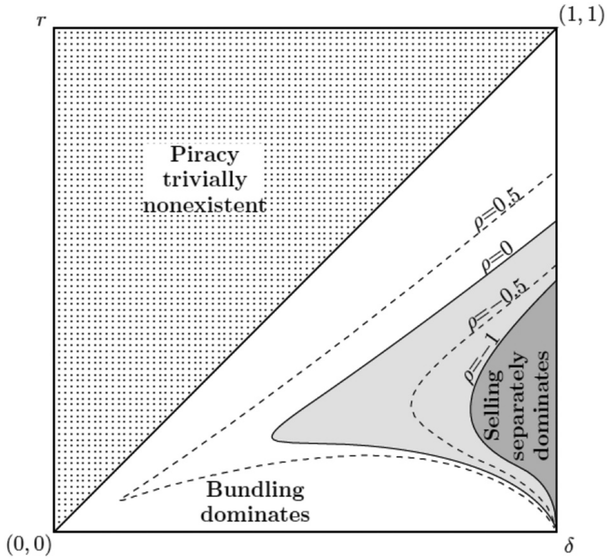

Figure 6. Optimality of Bundling in the Presence of Piracy and Correlated Valuations

The important takeaways are as follows. First, in reality, the correlation is unlikely to be perfect. For that matter, it is unlikely to be zero as well. The reality will likely fall somewhere between these extremes. Thus, in most situations, the region in which bundling is dominated will be somewhat larger than the darker region in Figure 6, but at the same time, smaller than the lighter one. For example, when $\rho = -0.5$, their shared boundary is exactly as marked in the figure. Second, a negative correlation by itself does not guarantee that bundling will be optimal over the entire parameter space, just as zero marginal costs do not. What is perhaps more interesting is that they, together, cannot guarantee the same—as long as there is piracy, manufacturers need to approach bundling with adequate caution.

Finally, our results get only stronger if v_1 and v_2 are positively correlated. For example, when the correlation is $\rho = 0.5$, the boundary will shift as shown in the figure, indicating a significant expansion of the region where bundling is dominated. As the correlation increases further, the region will continue to expand, and eventually fill up the entire transparent space that is now unshaded. This is expected. When the correlation is perfect and positive, the concept of bundling becomes moot, with our multi-product manufacturer effectively degenerating into a single-product one.

Other Considerations

Using numerical techniques, we now examine the applicability of our main insights to various settings of practical interest.

Ethical Consumers

Prior research has argued that consumers are not necessarily homogeneous in terms of the piracy cost. In particular, some consumers could face an extremely high piracy cost and behave in an ethical manner [3, 19]. Such ethical consumers will buy product $i \in \{1, 2\}$ if and only if $v_i \geq p_i$, and when offered a bundle, they will buy it if and only if $v_1 + v_2 \geq p$. Other consumers, the unethical ones, will act as described in the preceding sections.

To make our analysis realistic, we also consider the fact that high-valuation consumers are more likely to be ethical, especially since factors such as affluence and age can play a role in this regard. Accordingly, we assume the probability of a consumer being ethical to be $\eta\left(\frac{v_1+v_2}{2}\right)$, his average valuation of $\frac{v_1+v_2}{2}$ taken to be representative of his affluence. Figure 7 shows how our results change when $\eta = 0.5$, compared to our original model in which $\eta = 0$.

In Figure 7, there are two shaded regions, a darker region located inside a lighter one. The lighter one (inclusive of the darker region inside) is the same as the one in Figure 4. The darker region is where bundling is suboptimal, that is, for $\eta = 0.5$. Two observations are in order here. First, the region in which separate selling dominates still looks like a funnel, indicating that our earlier results are qualitatively robust. Second, the region is decreasing in η . This is intuitive—as we increase the size of the ethical segment, piracy becomes less potent and naturally less effective.

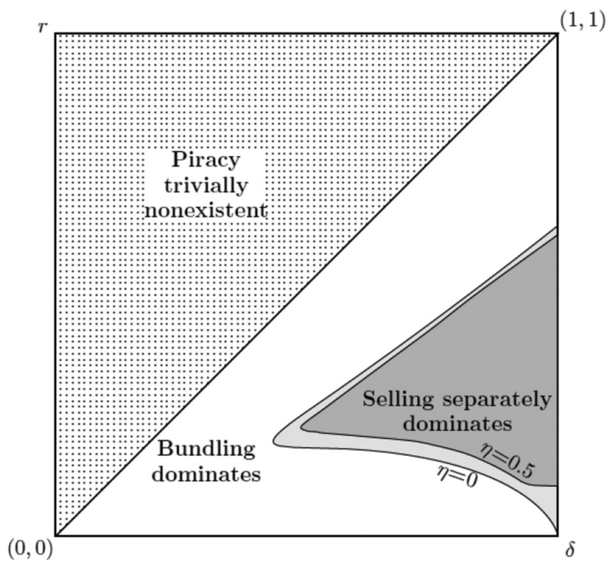

Figure 7. Optimality of Bundling in the Presence of Piracy with Ethical Consumers

Heterogeneity in Perceived Quality of Pirated Content

Different consumers might perceive the quality of the pirated product differently, which can make δ non-uniform across the consumer population. Accordingly, we now consider a setting where a $\nu > 0$ fraction of consumers faces $\frac{\delta}{2}$ while the rest, δ . Note that $\nu = 0$ corresponds to our original setup. Figure 8 shows how the results will change for $\nu = 0.5$.

The main insights remain similar despite this new heterogeneity. Further, compared to our original model, piracy is now less appealing to one half of the consumer market with the segment facing $\frac{\delta}{2}$, so its adverse effect on bundling is predictably smaller, just as is the region where separate selling now dominates.

Multiple Products

We now consider settings with more than two products, which are likely in practice. When the products are sold separately, the pricing problem remains separable as before. The bundling problem changes somewhat but stays analogous. Specifically, a consumer purchases an n -product bundle if and only if his valuations satisfy $\sum_{i=1}^{n} v_i - p \geq \sum_{i=1}^{n} (\delta v_i - r)^+$. In Figure 9, we show the results from our numerical analysis. The lighter region (inclusive of the darker regions contained inside) is where bundling is suboptimal in the two-product case. The darkest region represents where it is suboptimal in the four-product setting, and the medium-dark region (with the darkest region) corresponds to the three-product scenario.

As can be seen from the figure, the region where bundling is suboptimal shrinks with the bundle size. The intuition is as follows. The distribution of consumer valuations for the bundle becomes more concentrated near the mean as the number of products increases,

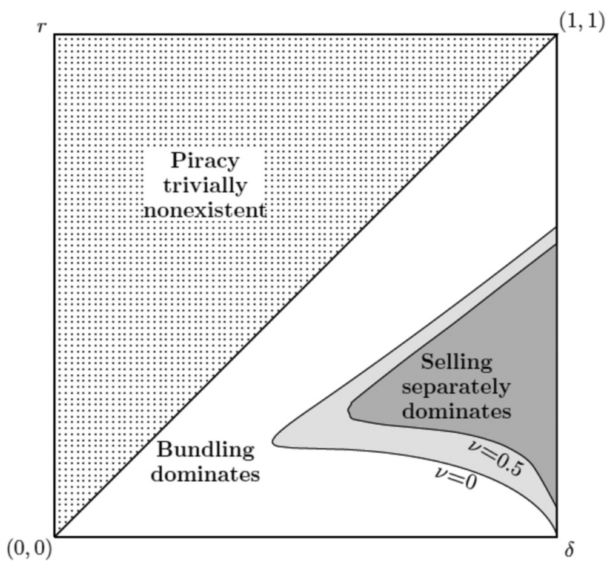

Figure 8. Optimality of Bundling in the Presence of Piracy and Consumer Heterogeneity in the Valuation for Pirated Content

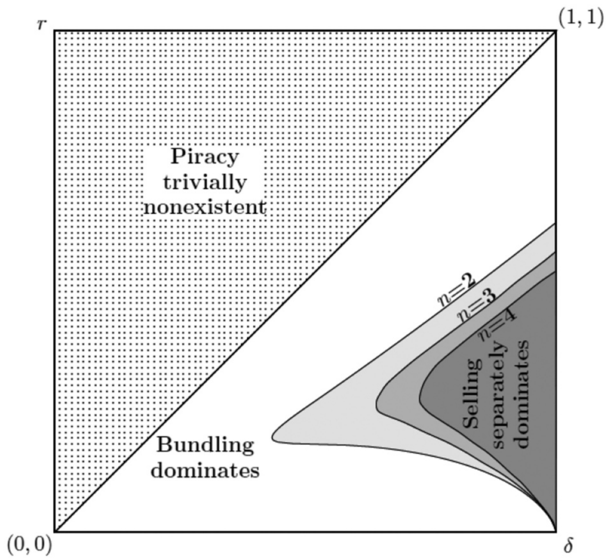

Figure 9. Optimality of Bundling in the Presence of Piracy in *n*-Product Scenario

making bundling a more effective price-discrimination tool. Accordingly, bundling dominates in a larger portion of the parameter space in a three-product scenario than it does in a two-product one, and the same argument extends to the four-product case as well.

Endogenous Piracy Cost

A manufacturer might lobby for more enforcement [19], making r endogenous. To analyze such a situation, we here model the cost of lobbying as κr^2 where $\kappa > 0$, and optimize $\Pi(r) - \kappa r^2$, where $\Pi(r)$ is the optimal revenue for a given r under the chosen strategy. If the manufacturer employs bundling, $\Pi(r) = p^* q(p^*)$ is determined from Proposition 1; otherwise, $\Pi(r)$ is the optimal profit under separate selling and is given by $2p_i^* q_i\left(p_i^*\right)$. Figure 10 shows the results from this new analysis. Evidently, the insight that a large δ makes selling separately preferable remains intact. Further, a high κ translates to a low r in equilibrium, and a low κ to a high r , which explains the shape of the shaded region.

Let us now compare the optimal value of r under bundling (denoted by r_B) with that under separate selling (denoted by r_S). This comparison is shown in Figure 11. Interestingly, $r_B > r_S$ does not hold in a large part of the parameter space. In other words, despite the fact piracy can significantly diminish the appeal of bundling, it is not necessarily true that the manufacturer wants a higher piracy cost when bundling. In general, when δ is high, the adverse effect of piracy on bundling is also high, and the manufacturer has not much to gain from investments in r_B . If κ decreases, however, the calculus starts favoring r_B a bit more, as the manufacturer can now invest to a point where piracy can be reasonably controlled.

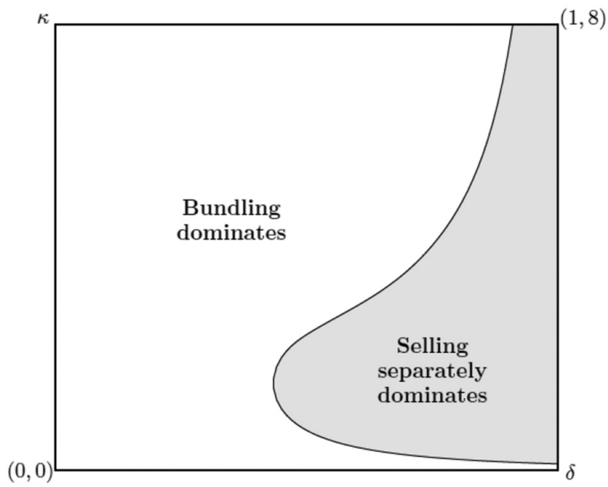

Figure 10. Optimality of Bundling When Piracy Cost Is Endogenous

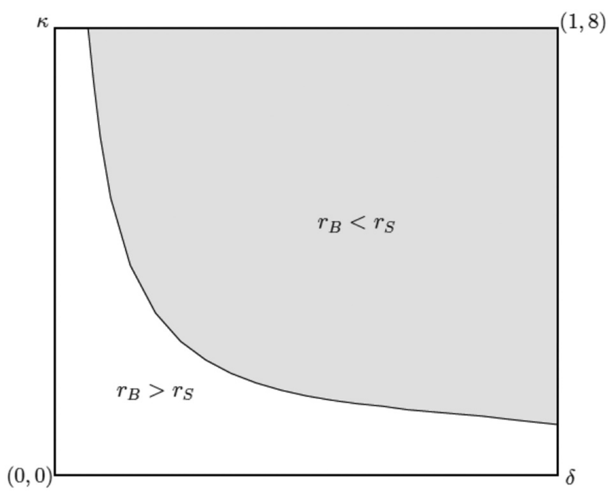

Figure 11. Comparison of Optimal Piracy Costs under Bundling and Separate Selling

It is important to note that, just as r can be endogenously chosen by a manufacturer, so can be δ , for example, by means of digital rights management technology. To address that possibility, we have also done an analysis that endogenizes δ based on a cost of $\hat{\kappa}(1-\delta)^2$, where $\hat{\kappa}>0$. This analysis too confirms the robustness of our main findings. Further, in almost all situations, the manufacturer prefers a lower δ when bundling, which is consistent with our earlier finding that bundling faces stronger headwinds from piracy at higher levels of δ .

Mixed Bundling

Can mixed bundling—the strategy of offering a bundle alongside individual products—retain its dominance in the presence of piracy even as pure bundling loses its appeal? To answer, let us first note that, when piracy is absent, it is indeed optimal for the manufacturer to use mixed bundling, with a price of $p_i^* = \frac{2}{3}$ for each product and $p^* = \frac{4-\sqrt{2}}{3}$ for the bundle. This results in a total revenue of approximately 0.549 . Recall that, if the manufacturer sells separately, it makes a profit of 0.5, and if it uses pure bundling, it makes approximately 0.544 , both less than what it makes from mixed bundling.

Under piracy, the bundle demand, q , can be estimated from the fact that a consumer purchases the bundle if and only if $v_1 + v_2 - p \geq \max\{v_1 - p_1, \delta v_1 - r, 0\} + \max\{v_2 - p_2, \delta v_2 - r, 0\}$. The demand for product i , q_i, can be determined from the fact that the consumer purchases i but not $j \neq i$ if and only if $v_i - p \geq (\delta v_i - r)^+$ and $v_j - (p - p_i) \leq (\delta v_j - r)^+$. The first inequality ensures that a consumer prefers purchasing product i to pirating or forgoing its use. The second guarantees that he would rather pirate product j , or even forgo its use, than buy the entire bundle. The manufacturer chooses p , p_1, and p_2 to maximize the total profit, $pq + p_1 q_1 + p_2 q_2$. Unfortunately, this optimization problem is not analytically tractable. So, to gain insights, we start with the case of $\delta \to 1$, when pirated products are nearly perfect substitutes for legal ones.

- **Proposition 4 (Mixed bundling solution and dominance of separate selling)**. *When $\delta \to 1$, the mixed bundling problem solution is as follows, and separate selling dominates all forms of bundling for $r \leq \frac{1}{3}$.*

$$\{p_i^*, p^*\} = \begin{cases} \{r, 2r\}, & \text{if } r \leq \frac{1}{3}, \\ \left\{r, \frac{4-\sqrt{2\left((3r-2)^2+1\right)}}{3}\right\}, & \text{if } \frac{1}{3} < r < \frac{2}{3}, \\ \left\{\frac{2}{3}, \frac{4-\sqrt{2}}{3}\right\} & \text{if } r \geq \frac{2}{3}. \end{cases}$$

Proposition 4 is instructive. It shows that, in the presence of piracy, mixed bundling is not necessarily the dominant strategy. In fact, when $r \leq \frac{1}{3}$, the manufacturer is better off choosing $p^* = p_1^* + p_2^*$, effectively abandoning mixed bundling in favor of separate selling. When r is large, however, the threat of piracy subsides, and mixed bundling regains its dominant status.

The result in Proposition 4 is also an indication that, even when δ is not very close to one, there could be a region where mixed bundling is no longer dominant. Using numerical experiments, we have been able to characterize this region; see Figure 12. Among the two shaded regions there, the darker one is where selling separately dominates all forms of bundling. In the lighter one, it dominates only pure bundling. The key takeaway is that our earlier results are relevant to manufacturers considering either form of bundling. Further, as the substitutability between legal and pirated versions increases, so do the potency of piracy and its impact on the efficacy of pure and mixed bundling. This is precisely why the shaded regions are the widest when δ is close to one, while both taper off gradually as δ goes down.

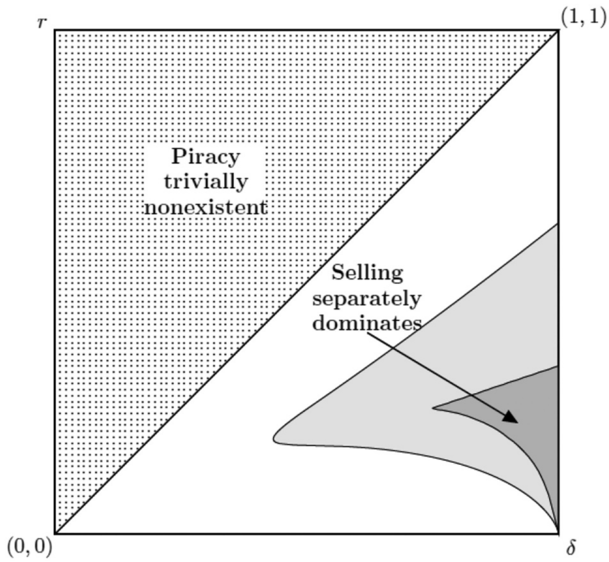

Figure 12. Optimality of Mixed Bundling in the Presence of Piracy

Conclusion

Given how widespread piracy is today and how broad its reach has become, it is important to find out whether bundling piracy-prone information goods is indeed a profitable strategy. To this end, we employed a parsimonious setup involving two key parameters: δ , the relative quality of the pirated product; and r , the cost of piracy. A summary of our analytical modeling-based theoretical results is included in Table 4.

We found that there is a sizable region in the (δ, r) space where bundling is dominated. The reason is as follows. The primary benefit of bundling is that it reduces the effective demand elasticity and, in so doing, enhances a monopolist's pricing power. However, raising the price is possible only if the monopolist is still in command of its monopoly power, which is certainly not the case when piracy is a serious threat. More specifically, piracy allows consumers to consume à la carte, while bundling does not. In other words, a consumer can selectively pirate the product of interest instead of spending a fortune on an entire bundle of products. This

Table 4. Summary of Analytical Modeling-Based Theoretical Findings

Findings	Description of findings	Related citations
Proposition 1	Provides the optimal bundling price under piracy and establishes that bundling is not necessarily effective in the presence of piracy; in fact, separate selling may be more profitable depending on the quality of pirated products and piracy cost	[4, 13]
Proposition 2	Shows that bundling would have been more profitable than separate selling if consumer were not allowed to pirate à la carte	[4, 13]
Proposition 3	Even if a strong negative correlation exists between valuations for bundle components, bundling would not necessarily be effective in the presence of piracy	[1, 4]
Proposition 4	Even mixed bundling may not outperform separate selling profit-wise when piracy is present	[1]
Extensions	The main results with respect to the sub-optimality of bunding are robust under various conditions (e.g., presence of ethical consumers, consumer heterogeneity in valuation for pirated goods, bundling of three or more products)	—

flexibility to consume selectively is, in fact, critical, and as we demonstrated, it creates an added incentive for piracy when a manufacturer employs bundling. And, in some cases, the losses from increased piracy can more than wipe out the purported benefits of bundling.

How widely applicable are these insights? We considered several model extensions and found that the results indeed hold in a variety of situations, for example, when valuations for the two products are correlated, when consumers are heterogeneous in their perception of the quality of pirated products, or when affluent users exhibit lesser proclivity for piracy. Not only that, our insights about the effect of piracy on pure bundling also extend qualitatively to mixed bundling.

Our findings have important implications. First, manufacturers should recognize that piracy can make bundling much less effective than commonly believed. Second, they should be particularly skeptical about the usefulness of bundling when pirated goods are considered close substitutes for legal ones. In other words, unless a manufacturer can find a way to sufficiently degrade the perceived quality of illegal alternatives to its products, it should refrain from bundling. This insight is practically relevant since, even in the absence of piracy, the gains from bundling are often modest. For example, in our two-product setting with uniform valuations, the profit rises by only 9% when bundling is adopted, and one cannot be confident that the modest gains will hold when piracy also becomes a part of the calculus.

Third, the impact of r , the piracy cost faced by the consumers, is far more nuanced and difficult to anticipate. If the cost is low, as it might be in some developing economies, the manufacturer should prefer bundling. In such a situation, piracy hurts separate selling and bundling alike and, therefore, does not impact their relative appeal in a significant way. When the cost is moderate, however, selling separately could surprisingly become optimal, particularly if the quality of pirated products is close to that of legal ones. This is because there are higher incentives to pirate in a selective manner. Finally, if the piracy cost is high—a situation that seems unlikely in practice—piracy will be a non-issue, and bundling would again be the dominant option.

Some caveats are necessary though. First, we did not consider commercial piracy [e.g., 23]. To be more specific, we ruled out scenarios in which there is a large pirate supplier operating as a profit-maximizing firm. Essentially, we took the market for illegal goods as fragmented, with pirate suppliers behaving as price-takers; this is consistent with empirical observations that there are often multiple sources supplying illegal goods and blocking one has little impact on consumers' choices [8]. Second, we assumed a monopoly setting, and it is not clear whether the insights from our work actually carry over to a competitive one. Although the monopoly assumption is reasonable in many practical contexts, the issue of competition could be important in certain others [e.g., 29]. While we leave the task of analyzing competition to future research, it would be remiss of us to fail to note that piracy essentially plays the role of a shadow competitor. In fact, piracy not only presents an alternative to the consumer, it also allows him to consume selectively when the manufacturer employs bundling.

As we have shown, this flexibility provided by piracy could significantly limit the gains from bundling and even render it suboptimal. Now, it is quite possible that, in a competitive market as well, a manufacturer would be hesitant to adopt bundling, especially if the competitors decide to sell a similar set of products separately. Thus, in the presence of competing manufacturers, the appeal of bundling should be further diminished and our findings with respect to its reduced efficacy would likely extend. Of course, we must still be careful when extrapolating our findings to intensely competitive settings. Intense competition among manufacturers could bring prices down to very low levels, driving piracy to extinction. Moreover, intense competition can unexpectedly increase the appeal of bundling [7]. Clearly, additional research is necessary to develop a better intuition into the dynamics of highly competitive markets.

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Disclosure Statement

No potential conflict of interest was reported by the authors.

Notes on contributors

Chen Jin is an assistant professor at the Department of Information Systems and Analytics, School of Computing, National University of Singapore. He received his Ph.D. from the Department of Industrial Engineering and Management Sciences, Northwestern University. Dr. Jin's research interests are primarily in the areas of information systems and operations interfaces. His papers have appeared in such journals as *Information Systems Research, Management Science*, and *Manufacturing & Service Operations Management*.

Chenguang (Allen) Wu is an assistant professor at the Department of Industrial Engineering and Decision Analytics, Hong Kong University of Science and Technology. He received his Ph.D. from the Department of Industrial Engineering and Management Sciences, Northwestern University. Dr. Wu's research interests include service operations as well as operations and information systems interfaces. His papers have appeared in such journals as *Management Science, Operations Research*, and *Manufacturing & Service Operations Management*.

Atanu Lahiri is an associate professor at the Jindal School of Management, University of Texas at Dallas. He received his Ph.D. from the Simon School of Business, University of Rochester. His research interests are at the intersection of information systems and economics. Dr. Lahiri's work has appeared in journals such as *Information Systems Research, Journal of Management Information Systems, Management Science, MIS Quarterly*, and *INFORMS Journal on Computing*. He serves as an associate editor for *Information Systems Research*.

ORCID

Chen Jin http://orcid.org/0000-0001-9940-0757
Chenguang (Allen) Wu http://orcid.org/0000-0002-2528-0286
Atanu Lahiri http://orcid.org/0000-0003-3174-8014

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Appendix: Proofs

Proof of Proposition 1. The manufacturer solves $\max_p pq(p)$ where $q(p)$ is as described in (A1). Case (i), $0 \le r < \frac{\delta(1-\delta)}{2-\delta}$:

$$q(p) = \begin{cases} 1-\frac{p^2}{2}, & \text{if } 0 \le p \le \frac{r}{\delta}, \\ \frac{1}{2}\left(\frac{2r}{\delta}-p\right)^2 + \left(1-\frac{r}{\delta}\right)^2 + 2\left(1-\frac{r}{\delta}\right)\frac{r}{\delta} - \left(p-\frac{r}{\delta}\right)\left(\frac{p-r}{1-\delta}-\frac{r}{\delta}\right), & \text{if } \frac{r}{\delta} < p \le \frac{2r}{\delta}, \\ \left(1-\frac{r}{\delta}\right)^2 - \frac{1}{2}\left(\frac{p-2r}{1-\delta}-\frac{2r}{\delta}\right)^2 + \left(2-\frac{2p-3r}{1-\delta}+\frac{r}{\delta}\right)\frac{r}{\delta}, & \text{if } \frac{2r}{\delta} < p \le 1-\delta+r, \\ \left(1-\frac{r}{\delta}\right)^2 - \frac{1}{2}\left(\frac{p-2r}{1-\delta}-\frac{2r}{\delta}\right)^2 + \left(1-\frac{p-2r}{1-\delta}+\frac{r}{\delta}\right)\left(\frac{r}{\delta}-p+r+1-\delta\right), & \text{if } 1-\delta+r < p \le 1-\delta+r+\frac{r}{\delta}, \\ \frac{1}{2}\left(2-\frac{p-2r}{1-\delta}\right)^2, & \text{if } 1-\delta+r+\frac{r}{\delta} < p \le 2(1-\delta+r). \end{cases} \quad (A1)$$

Case (ii), $\frac{\delta(1-\delta)}{2-\delta} \le r < \delta$:

$$q(p) = \begin{cases} 1-\frac{p^2}{2}, & \text{if } 0 \le p \le \frac{r}{\delta}, \\ \frac{1}{2}\left(\frac{2r}{\delta}-p\right)^2 + \left(1-\frac{r}{\delta}\right)^2 + 2\left(1-\frac{r}{\delta}\right)\frac{r}{\delta} - \left(p-\frac{r}{\delta}\right)\left(\frac{p-r}{1-\delta}-\frac{r}{\delta}\right), & \text{if } \frac{r}{\delta} < p \le 1-\delta+r, \\ \frac{1}{2}\left(\frac{2r}{\delta}-p\right)^2 + \left(1-\frac{r}{\delta}\right)^2 + \left(\frac{3r}{\delta}-2p+r+1-\delta\right)\left(1-\frac{r}{\delta}\right), & \text{if } 1-\delta+r < p \le \frac{2r}{\delta}, \\ \left(1-\frac{r}{\delta}\right)^2 - \frac{1}{2}\left(\frac{p-2r}{1-\delta}-\frac{2r}{\delta}\right)^2 + \left(1-\frac{p-2r}{1-\delta}+\frac{r}{\delta}\right)\left(\frac{r}{\delta}-p+r+1-\delta\right), & \text{if } \frac{2r}{\delta} < p \le 1-\delta+r+\frac{r}{\delta}, \\ \frac{1}{2}\left(2-\frac{p-2r}{1-\delta}\right)^2, & \text{if } 1-\delta+r+\frac{r}{\delta} < p \le 2(1-\delta+r). \end{cases}$$

We only consider what happens if $r < \delta$, as the case of $r \ge \delta$ is the same as our benchmark model in which there is no piracy. Further, note that $\frac{p}{2}\left(2-\frac{p-2r}{1-\delta}\right)^2$ is maximized at $2(1-\delta+r)/3$, which is less than $1-\delta+r+r/\delta$. Therefore, we can ignore the last sub-case of Case (i) as well as that of Case (ii). We need to analyze only the remaining sub-cases in (A1), henceforth referred to as Cases (i)(1) – (i)(4) and (ii)(1) – (ii)(4). We will use the notation p_{ij} to denote the interior maximum for a sub-case; for example, for Case (i)(3), we will call it p_{13}.

Case (i)(1): $0 \le p \le r/\delta$. We denote the revenue by $F_{11}(p) \triangleq p(1-p^2/2)$, which achieves its maximum at $p_{11} = \sqrt{2/3}$. According to the definition of Case (i), this solution is valid if $p_{11} \le r/\delta$, that is, if $r \ge \delta\sqrt{2/3}$. However, Case (i) also requires $r < \frac{\delta(1-\delta)}{2-\delta}$, which is impossible to satisfy when $r \ge \delta\sqrt{2/3}$. So, Case (i)(1) can be ruled out as well.

Case (i)(2): $\frac{r}{\delta} < p \le \frac{2r}{\delta}$. The revenue function in this case is:

$$F_{12}(p) \triangleq p\left(\frac{1}{2}\left(\frac{2r}{\delta}-p\right)^2 + \left(1-\frac{r}{\delta}\right)^2 + 2\left(1-\frac{r}{\delta}\right)\frac{r}{\delta} - \left(p-\frac{r}{\delta}\right)\left(\frac{p-r}{1-\delta}-\frac{r}{\delta}\right)\right),$$

which is maximized at $p_{12} = \frac{4r+\sqrt{2\left(3-3r^2/\delta+5r^2-3\delta^2\right)}}{3(1+\delta)}$. Note that $3-3r^2/\delta+5r^2-3\delta^2$ is linear in r^2 and positive for $r \in \left(0, \frac{\delta(1-\delta)}{2-\delta}\right)$. According to (A1), we require $p_{12} \le 2r/\delta$. This condition is equivalent to $r \ge \delta\sqrt{\frac{1-\delta}{6-\delta}}$. We also need $p_{12} > r/\delta$, which translates to $r < \delta\sqrt{2/3}$.

Case (i)(3): $2r/\delta < p \le 1-\delta+r$. Following (A1), we define the revenue as:

$$F_{13}(p) \triangleq p\left(\left(1-\frac{r}{\delta}\right)^2 - \frac{1}{2}\left(\frac{p-2r}{1-\delta}-\frac{2r}{\delta}\right)^2 + \left(2-\frac{2p-3r}{1-\delta}+\frac{r}{\delta}\right)\frac{r}{\delta}\right),$$

which is maximized at $p_{13}=\frac{4r+\sqrt{2\left(3\delta^2-6\delta+5r^2+3-3r^2/\delta\right)}}{3}$. Note that $3\delta^2-6\delta+5r^2+3-3r^2/\delta$ is linear in r^2 and positive for $r\in\left(0,\frac{\delta(1-\delta)}{2-\delta}\right)$. Now, $p_{13}>2r/\delta$ is equivalent to $r<\delta\sqrt{\frac{1-\delta}{6-\delta}}$. As far as the relation between p_{13} and $1-\delta+r$ is concerned,

$$p_{13}<1-\delta+r\Leftrightarrow r^2(3-2/\delta)+2r(1-\delta)-(1-\delta)^2\leq 0.$$

When $\delta\in(0,2/3]$, one can verify that the last inequality indeed holds. When $\delta\in(2/3,\ 1)$, we need to consider the roots of $r^2(3-2/\delta)+2r(1-\delta)-(1-\delta)^2=0$: $r_a=\frac{\delta(1-\delta)}{3\delta-2}\left(\sqrt{4-2/\delta}-1\right)$ and $r_b=-\frac{\delta(1-\delta)}{3\delta-2}\left(\sqrt{4-2/\delta}+1\right)$. Now, $\frac{\delta(1-\delta)}{2-\delta}\geq r_a>0>r_b$, and $p_{13}\leq 1-\delta+r$ if and only if $r\leq r_a$.

Case (i)(4): $1-\boldsymbol{\delta}+\boldsymbol{r}<\boldsymbol{p}\leq 1-\boldsymbol{\delta}+\boldsymbol{r}+\frac{\boldsymbol{r}}{\boldsymbol{\delta}}$. The revenue function for this sub-case is as follows:

$$F_{14}(p)\triangleq p\left(\left(1-\frac{r}{\delta}\right)^2-\frac{1}{2}\left(\frac{p-2r}{1-\delta}-\frac{2r}{\delta}\right)^2+\left(1-\frac{p-2r}{1-\delta}+\frac{r}{\delta}\right)\left(\frac{r}{\delta}-p+r+1-\delta\right)\right).$$

When $\delta\in(0,\ 2/3]$, $\frac{dF_{14}(p)}{dp}<0$ at $p=1-\delta+r$, implying that no valid interior solution is possible. When $\delta\in(2/3,\ 1)$, the interior maximum becomes $p_{14}=\frac{4\delta r+8\delta-4-4\delta^2+A}{3(2\delta-1)}$, where $A>0$ is as in Table 2 (main article). This maximum abides by the validity conditions if $r_a\leq r<\frac{\delta(1-\delta)}{2-\delta}$.

Case (ii)(1): $\boldsymbol{0}\leq\boldsymbol{p}\leq\frac{\boldsymbol{r}}{\boldsymbol{\delta}}$. As in Case (i)(1), the interior maximum is $p_{21}=p_{11}=\sqrt{2/3}$, so $p_{21}\leq r/\delta$ is equivalent to $r\geq\delta\sqrt{2/3}$.

Case (ii)(2): $\frac{\boldsymbol{r}}{\boldsymbol{\delta}}<\boldsymbol{p}\leq 1-\boldsymbol{\delta}+\boldsymbol{r}$. The revenue for this sub-case, $F_{22}(p)$, is the same as that in Case (i)(2). So, the interior maximum, p_{22}, is identical to p_{12}. Now, $p_{22}>r/\delta$ is equivalent to $r<\delta\sqrt{2/3}$, and $p_{22}\leq 1-\delta+r$ is equivalent to $D\triangleq r^2\left(10-6/\delta-(1-3\delta)^2\right)+6(1-\delta)-6(3\delta-1)\left(1-\delta^2\right)r-9\left(1-\delta^2\right)^2\leq 0$, which holds for all $r\in[\frac{\delta(1-\delta)}{2-\delta},\ \delta)$ when $\delta\in(0,\ 2/3]$. When $\delta\in(2/3,\ 1)$, we can solve $D=0$ to get the following roots: $r_3=\frac{\delta}{3\delta-2}\left(3\delta-1+\sqrt{4-2/\delta}\right)$ and $r_4=\frac{\delta}{3\delta-2}\left(3\delta-1-\sqrt{4-2/\delta}\right)$. One can verify that $r_3>\delta>r_4$ and that $D\leq 0$ if and only if $r\geq r_4$

Case (ii)(3): $1-\boldsymbol{\delta}+\boldsymbol{r}<\boldsymbol{p}\leq 2\boldsymbol{r}/\boldsymbol{\delta}$. The revenue in this case is:

$$F_{23}(p)\triangleq p\left(\frac{1}{2}\left(\frac{2r}{\delta}-p\right)^2+\left(1-\frac{r}{\delta}\right)^2+\left(\frac{3r}{\delta}-2p+r+1-\delta\right)\left(1-\frac{r}{\delta}\right)\right),$$

which is maximized at $p_{23}=\left(4-\sqrt{2(3\delta-6r+2+3r^2/\delta)}\right)/3$. Now, $p_{23}>1-\delta+r$ is never satisfied when $\delta\in(0,\ 2/3]$, implying that this sub-case is not relevant for $\delta\leq 2/3$. When $\delta\in(2/3,\ 1)$, however, $p_{23}>1-\delta+r$ if and only if $r<r_4$, where r_4 is as defined above under Case (ii)(2). Moving on to the other validity condition, $p_{23}\leq 2r/\delta$, it is equivalent to $r^2(\delta-6)-2\delta r(\delta-4)-\delta^2(2-\delta)\geq 0$. When $\delta>2/3$, to determine validity, we need to consider the roots of $r^2(\delta-6)-2\delta r(\delta-4)-\delta^2(2-\delta)=0$: $r_5=\delta$ and $r_6=\frac{\delta(2-\delta)}{6-\delta}$; only r_6 is of interest and $p_{23}\leq 2r/\delta$ holds if and only if $r\geq r_6$.

Case (ii)(4): $2\boldsymbol{r}/\boldsymbol{\delta}<\boldsymbol{p}\leq 1-\boldsymbol{\delta}+\boldsymbol{r}+\boldsymbol{r}/\boldsymbol{\delta}$. The demand function in this case, $F_{24}(p)$, is the same as that in Case (i)(4), $F_{14}(p)$. So, the interior maximum, p_{24}, is identical to p_{14}. Since the derivative of $F_{24}(p)$ at $p=1-\delta+r+r/\delta$ is non-positive, we must have $p_{24}\leq 1-\delta+r+r/\delta$. Likewise, from the sign of the derivative at $p=2r/\delta$, we can confirm that, when $\delta\leq 2/3$, $p_{24}\leq 2r/\delta$ is automatically satisfied. When $\delta>2/3$, however, $p_{24}>2r/\delta$ if and only if $r<r_6$, where r_6 is as defined above under Case (ii)(3).

Finally, to combine all the sub-cases into the desired expression for p^*, we just need to note that, when $\delta\in(2/3,\ 1)$, $r_a<\frac{\delta(1-\delta)}{2-\delta}<r_6<r_4<\delta\sqrt{2/3}$. So, as we increase r from 0 to δ and cross these five thresholds one by one, we transition through six different cases in the following order: Case (i)(3), Case (i)(4), Case (ii)(4), Case (ii)(3), Case (ii)(2), and Case (ii)(1). In contrast, when $\delta \in (0,\ 2/3]$, the relevant thresholds are ordered as follows: $\delta\sqrt{\frac{1-\delta}{6-\delta}} \le \frac{\delta(1-\delta)}{2-\delta} < \delta\sqrt{\frac{2}{3}}$; therefore, we would transition through Case (i)(3), Case (i)(2), Case (ii)(2), and Case (ii)(1) in this specific order.

Proof of Proposition 2. Consumers who satisfy $v_1 + v_2 - p \ge (\delta(v_1 + v_2) - 2r)^+$ buy the bundle. The resulting demand can be expressed as follows:

Case (i), $0 \le r/\delta < 1/2$:

$$q(p) = \begin{cases} 1 - \frac{p^2}{2}, & \text{if } 0 \le p < \frac{2r}{\delta}, \\ 1 - \frac{1}{2}\left(\frac{p-2r}{1-\delta}\right)^2, & \text{if } \frac{2r}{\delta} \le p < 1 - \delta + 2r, \\ \frac{1}{2}\left(2 - \frac{p-2r}{1-\delta}\right)^2, & \text{if } 1 - \delta + 2r \le p < 2(1 - \delta + r). \end{cases}$$

Case (ii), $1/2 \le r/\delta < 1$:

$$q(p) = \begin{cases} 1 - \frac{p^2}{2}, & \text{if } 0 \le p < 1, \\ \frac{(p-2)^2}{2}, & \text{if } 1 \le p < \frac{2r}{\delta}, \\ \frac{1}{2}\left(2 - \frac{p-2r}{1-\delta}\right)^2, & \text{if } \frac{2r}{\delta} \le p < 2(1 - \delta + r). \end{cases}$$

The producer's problem is to maximize $pq(p)$. Let us start with Case (i). Note that $p(1 - p^2/2)$ is maximized at $p = \sqrt{2/3}$. This solution is valid if $p < 2r/\delta$ of if $r > (\delta/2)\sqrt{2/3}$. Similarly, maximizing $p\left(1 - \frac{1}{2}\left(\frac{p-2r}{1-\delta}\right)^2\right)$ leads to $p = \left(4r + \sqrt{6(1-\delta)^2 + 4r^2}\right)/3$, which is larger than $2r/\delta$ if $r \le \frac{\delta}{2}\sqrt{\frac{2(1-\delta)}{3-\delta}}$. Between these two bounds, or if $\frac{\delta}{2}\sqrt{\frac{2(1-\delta)}{3-\delta}} < r \le \frac{\delta}{2}\sqrt{\frac{2}{3}}$, the corner solution of $p = 2r/\delta$ applies. Finally, $\frac{p}{2}\left(2 - \frac{p-2r}{1-\delta}\right)^2$ is maximized at $p = 2(1 - \delta + r)/3$, but this solution is always smaller than $1 - \delta + 2r$ and thus cannot be valid. Taken together, we can write:

$$p^* = \begin{cases} \frac{4r + \sqrt{6(1-\delta)^2 + 4r^2}}{3}, & \text{if } 0 \le r \le \frac{\delta}{2}\sqrt{\frac{2(1-\delta)}{3-\delta}}, \\ \frac{2r}{\delta}, & \text{if } \frac{\delta}{2}\sqrt{\frac{2(1-\delta)}{3-\delta}} < r \le \frac{\delta}{2}\sqrt{\frac{2}{3}}, \\ \sqrt{\frac{2}{3}}, & \text{if } \frac{\delta}{2}\sqrt{\frac{2}{3}} < r < \frac{\delta}{2}. \end{cases}$$

Moving on to Case (ii), as before, $p(1 - p^2/2)$ is maximized at $p = \sqrt{2/3}$. Now, $p(2-p)^2/2$ is maximized at $p = 2/3$, but this solution cannot be valid since it does not satisfy $p \ge 1$. Once again, $\frac{p}{2}\left(2 - \frac{p-2r}{1-\delta}\right)^2$ is maximized at $p = 2(1 - \delta + r)/3$, and again it is not valid because $1/2 \le r/\delta < 1$ implies that $2(1 - \delta + r)/3 < 2r/\delta$. Taken together, $p = \sqrt{2/3}$ is optimal when $r \ge \delta/2$.

Finally, we can easily combine the solutions for Cases (i) and (ii) to get the desired expression for p^*. It is easy to verify that the resulting bundle revenue is higher than $1/2$ at all r and δ.

Proof of Proposition 3. The bundle demand as a function of price is as follows.

$$\textit{Case}(i),\ 0 \le r < \tfrac{\delta}{2} : q(p) = \begin{cases} 1, & \text{if } 0 \le p < 1 - \delta + r, \\ (2(1 - p + r) - \delta)/\delta, & \text{if } 1 - \delta + r \le p \le 1 - \delta + 2r. \end{cases}$$

$$\text{Case (ii)},\ \delta/2 \le r < \delta : q(p) = \begin{cases} 1, & \text{if } 0 \le p < 1 - \delta + r, \\ (2(1 - p + r) - \delta)/\delta, & \text{if } 1 - \delta + r \le p \le 1. \end{cases}$$

Consider Case (i) first. Using the first order condition, we can maximize the revenue, $pq(p)$, for the sub-case in which $1 - \delta + r \le p < 1 - \delta + 2r$. The interior maximum, p^* , obtained from the first-order condition, is $(1 + r)/2 - \delta/4$. If this p^* is between $1 - \delta + r$ and $1 - \delta + 2r$, it would indeed be optimal. Otherwise, if p^* is greater than $1 - \delta + 2r$, the optimal price would simply be $1 - \delta + 2r$. Likewise, if it is below $1 - \delta + r$, the optimal price would be $1 - \delta + r$. Hence,

$$p^* = \begin{cases} 1-\delta+2r, & \text{if } 0 \le r < \delta/2 - 1/3, \\ (1+r)/2 - \delta/4, & \text{if } \delta/2 - 1/3 \le r < 3\delta/2 - 1, \\ 1-\delta+r, & \text{if } r \ge 3\delta/2 - 1. \end{cases}$$

Since $3\delta/2 - 1 > 0$ is equivalent to $\delta > 2/3$, we can rewrite the above expression as follows.

$$p^* = \begin{cases} 1-\delta+r, & & \text{if } \delta \le 2/3, \\ 1-\delta+2r, & \text{if } 0 \le r < \delta/2 - 1/3, & \\ (1+r)/2 - \delta/4, & \text{if } \delta/2 - 1/3 \le r < 3\delta/2 - 1, & \text{if } \delta 2/3. \\ 1-\delta+r, & \text{if } r \ge 3\delta/2 - 1 & \end{cases}$$

Case (ii) can be solved in a similar fashion. Here, the interior solution of $(1+r)/2 - \delta/4$ is valid only if it is above $1-\delta+r$. Therefore:

$$p^* = \begin{cases} (1+r)/2 - \delta/4, & \text{if } 0 \le r < 3\delta/2 - 1, \\ 1-\delta+r, & \text{if } 3\delta/2 - 1 \le r < \delta, \\ 1, & \text{otherwise,} \end{cases}$$

which can be rewritten as

$$p^* = \begin{cases} 1-\delta+r, & \text{if } r < \delta, & \text{if } \delta \le 2/3, \\ 1, & \text{otherwise,} & \\ (1+r)/2 - \delta/4, & \text{if } 0 \le r < 3\delta/2 - 1, & \\ 1-\delta+r, & \text{if } 3\delta/2 - 1 \le r < \delta, & \text{if } \delta > 2/3. \\ 1, & \text{otherwise.} & \end{cases}$$

We can now easily combine Cases (i) and (ii) to get the desired expression for p^*.

Proof of Proposition 4. When $\delta \to 1$, a consumer purchases the bundle if and only if this holds:

$$v_1 + v_2 - p \ge \max\{v_1 - p_1, v_1 - r, 0\} + \max\{v_2 - p_2, v_2 - r, 0\}.$$

To ensure that the demand for the bundle is positive, we need $p \le \min\{p_1, r\} + \min\{p_2, r\}$, which further implies that $r \ge p - p_1$ and $r \ge p - p_2$.

Now, a consumer purchases product i alone if and only if the following holds:

$$v_i - p_i \ge (v_i - r)^+ \text{ and } v_j - (p - p_i) \le \left(v_j - r\right)^+, \; j \ne i.$$

For this demand function to be positive, we need $p_i \le r$.

Based on the conditions above, the demand for the bundle would be as follows:

$$\begin{aligned} q &= P[v_1 + v_2 - p \ge \max\{v_1 - p_1, v_1 - r, 0\} + \max\{v_2 - p_2, v_2 - r, 0\}] \\ &= P[v_2 \ge p - p_1, v_1 \ge p - p_2, v_1 + v_2 \ge p] \\ &= -\left(p_1^2 + p_2^2\right)/2 + p_1 + p_2 - 2p + 1 + p^2/2. \end{aligned}$$

And the demand for product $i \in \{1, 2\}$ would be:

$$\begin{aligned} q_i &= \mathbb{P}\left\{v_i - p_i \ge (v_i - r)^+, \; v_j - (p - p_i) \le \left(v_j - r\right)^+\right\} \\ &= \mathbb{P}\left\{v_i \ge p_i, \; v_j \le \min\{p - p_i, \; r\}\right\} = (1 - p_i)(p - p_i). \end{aligned}$$

Based on q_1, q_2, and q , the manufacturer's profit can be expressed as:

$$\Pi(p_1, p_2, p) = p_1(1 - p_1)(p - p_1) + p_2(1 - p_2)(p - p_2) + p\left(p_1 + p_2 - 2p + 1 + \frac{p^2}{2} - \frac{p_1^2 + p_2^2}{2}\right).$$

Let p_1^*, p_2^*, and p^* be the optimal prices obtained by maximizing $\Pi(p_1, p_2, p)$. It is easy to show $p_1^* = p_2^*$, which is not surprising given the symmetrical nature of the setting. Accordingly, we set both prices to z and rewrite the profit as follows.

$$\Pi(z,p) = 2z(1-z)(p-z) + p(1-z)(1+z-p) + \frac{p(2-p)(2z-p)}{2}.$$

Note that when $p = 2z$, mixed bundling effectively disappears, that is, it degenerates into separate selling. At the other extreme, when $z = p$, mixed bundling degenerates into pure bundling. So, we need to focus on the range $z \le p \le 2z$. The first-order condition gives us:

$$\frac{\partial \Pi(z,p)}{\partial p} = 0 \Rightarrow \begin{cases} p_{B1} = \left(4 - \sqrt{2\left(1+(3z-2)^2\right)}\right)/3, \\ p_{B2} = \left(4 + \sqrt{2\left(1+(3z-2)^2\right)}\right)/3. \end{cases}$$

It is easy to verify that p_{B1} is a maximum and p_{B2} a minimum. Therefore, p_{B1} is the root of interest. Now, if $z < 2/3$, $p_{B1} < 2z$ is equivalent to $(1-z)(3z-1) > 0$, which requires that $z > 1/3$. If $z \ge 2/3$, $p_{B1} < 2z$ obviously holds. Moreover,

$$p_{B1} > z \Leftrightarrow 4 - 3z > \sqrt{2\left(1+(3z-2)^2\right)} \Leftrightarrow z < \sqrt{2/3}.$$

Next, let $\hat{p}(z)$ be the optimal bundle price given $z \in [0,1]$. Since $z \le \hat{p}(z) \le 2z$, we should have:

$$\hat{p}(z) = \begin{cases} 2z, & \text{if } 0 \le z \le 1/3, \\ p_{B1} = \left(4 - \sqrt{2\left(1+(3z-2)^2\right)}\right)/3, & \text{if } 1/3 < z < \sqrt{2/3}, \\ z, & \text{if } \sqrt{2/3} \le z < 1. \end{cases}$$

Substituting the $\hat{p}(z)$ into $\Pi(z,p)$, we can rewrite the profit as a function of z :

$$\Pi(z,\hat{p}(z)) = \begin{cases} 2z(1-z), & \text{if } 0 \le z \le 1/3, \\ 16z/3 - 6z^2 + 2z^3 + \left(2\left(1+(3z-2)^2\right)\right)^{3/2}/27 - 28/27, & \text{if } 1/3 < z < \sqrt{2/3}, \\ z(1-z^2/2), & \text{if } \sqrt{2/3} \le z < 1. \end{cases}$$

It is easy to verify that: (i) $\Pi(z,\hat{p}(z))$ is continuous in z; (ii) $\Pi(z,\hat{p}(z))$ is increasing for $0 \le z < 2/3$; and (iii) $\Pi(z,\hat{p}(z))$ is decreasing for $z > 2/3$. So, the ideal choice for z is $2/3$. However, since any choice of z must abide by $z \le r$, the optimal solution is $z^* = \min\{2/3, r\}$. Substituting z^* into $\hat{p}(z)$, we get:

$$p^* = \begin{cases} 2r, & \text{if } 0 < r \le 1/3, \\ \left(4 - \sqrt{2\left(1+(3r-2)^2\right)}\right)/3, & \text{if } 1/3 < r < 2/3, \\ \left(4-\sqrt{2}\right)/3, & \text{if } r \ge 2/3. \end{cases}$$

From the optimal product price $p_1^* = p_2^* = z^*$ and the optimal bundle price p^*, the optimal mixed bundling profit can be easily determined. The rest follows from a straightforward comparison of this profit outcome with those for pure bundling and separate selling.