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Bundling Variety, Usage, or Both? A Multi-Service Analysis of Pay-Per-Use and Subscription Pricing

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Abstract

We depart from the classic bundling literature on single-unit purchases and develop a multi-unit demand model in which customers decide both the variety and volume of their purchased products. We integrate product bundling with two commonly used usage-based pricing schemes in the service industry, pay-per-use and subscriptions, and examine pricing tactics that span both variety and usage. Our model captures customers' diminishing margins of consumption, and we demonstrate an intricate interplay between product variety and usage that enriches existing results on the dominance of pure bundling over component selling in the classic single-unit demand model. Specifically, with multi-unit demands, component selling can outperform pure bundling under pay-per-use, whereas the reversal is true under subscriptions. We also analyze mixed bundling (on either variety or usage) and nonlinear pricing, and demonstrate how to leverage these more advanced pricing schemes for better revenues. Our results provide novel insights into firms' bundling strategies jointly on variety and usage. Collectively, they highlight the critical role of the customer demand model (single-unit vs. multi-unit) in driving monopoly firms' strategic choice of product (un)bundling.

Keywords

Bundling, pay-per-use, subscription, variety, usage, nonlinear pricing

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1 Introduction

For a number of service-oriented businesses such as telecommunications, entertainment, and rentals, customers' demands are carried out in a repeated manner and thus measured by usage. This prompts two usage-based pricing schemes in practice, *pay-per-use* (PPU) and *subscription*, differentiated by how usage is charged: pay-per-use charges on-demand usage, whereas subscriptions charge a one-time fee to provide unlimited access. For instance, a data provider may charge users based on the amount of data usage, but a gym may offer members unlimited access to its equipment, facilities, and court space.

Notice also that when a service firm offers more than one service, it may lump multiple services together into a single package, the practice known as *bundling* widely used in the travel, healthcare, and software industries. For example, spa shops may offer a package of beauty and wellness services, and auto shops may provide a bundle of maintenance and repair services. Despite its prevalence, less is known about the efficacy of bundling in settings where customers have multi-unit demands of a product or service. In these settings, it is natural to ask: How should a service firm devise pricing strategies that involve both *variety* and *usage*? Should the firm adopt pay-per-use or subscriptions in the usage dimension, in conjunction with bundling or unbundling in the variety dimension?

The answers to these questions are of practical significance. For example, Homie, a Netherlands-based company that operates rental businesses for appliances such as washers and dryers, chooses to unbundle different appliance components while charging renters on their actual usage. SUPS.com,

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Table 1. Practice of multi-service pricing under multi-unit demands.

Company	Industry	Pricing
Homie	Appliance	Component selling + pay-per-use
SUPS.com	Nutrients	Pure bundling + pay-per-use
Alibaba's 88VIP	E-Commerce	Pure bundling + subscriptions
Universal Orlando	Entertainment	Mixed bundling + pay-per-use & subscriptions
Ocean Park (Hong Kong)	Entertainment	Component Selling + pay-per-use & subscriptions
CityPass (New York)	Travel	Pure bundling + pay-per-use

a vitamin and supplement seller, bundles some of its nutrition products (e.g., Rise and Balance) and charges users monthly fees. Alibaba's "88VIP" is an annual subscription program that bundles numerous memberships of self-owned businesses, ranging from video streaming Youku, on-demand delivery Ele.me, to online ticketing Taopiaopiao. Most amusement parks sell a combination of annual and daily passes, but they vary in whether allowing visitors to enter different parks on the same day. See Table 1 for different implementations of multi-service pricing in various industries.

These observations hint at an intricate interplay between product variety and usage in service-oriented businesses. To study this interplay, we develop a multi-unit demand model that captures customers' diminishing margins of consumption and examine pricing strategies that involve both variety and usage. Unlike the product bundling literature with an emphasis on single-unit demands, customers in our setting must decide both the *variety* and *volume* of their purchased products or services. This ramification allows us to establish rich results that complement the conventional wisdom on product bundling. Specifically, we show that the effectiveness of bundling depends intimately on the price decisions adopted in the usage dimension.

To develop our results, we first fix pay-per-use in the usage dimension and analyze component selling, pure bundling, and mixed bundling with price variations in the variety dimension. Under pay-per-use, we show that the well-known dominance of pure bundling over component selling in the classic single-unit demand model is generally reversed. We explain such reversal by identifying a novel *compensation effect* that uniquely exists under pure bundling in the presence of multi-unit demands. This effect arises as customers are forced to buy the entire bundle only to access their preferred service. Thus, it refers to the fact that the benefit of purchasing one's preferred service must compensate for the disutility of purchasing her less preferred service. This weakens customers' incentives of purchasing their preferred service in the first place and has an adverse effect on bundle sales. In view of this effect, we propose mixed bundling as a remedy and we show that under mixed bundling, each customer will first purchase a base amount of the bundle and then an additional amount of her preferred service. This allows mixed bundling to eliminate the inconsistency between customers' purchase and consumption decisions for their less preferred service. As a result, mixed bundling can dominate both component selling and pure bundling under the pay-per-use scheme.

We next fix subscription pricing in the usage dimension and analyze component selling, pure bundling, and mixed bundling on variety. We find that the aforementioned compensation effect under pay-per-use vanishes under all subscription schemes as customers can fully customize their consumption plans under subscriptions. It then follows that, under subscriptions, pure bundling dominates component selling, and they both are dominated by mixed bundling. We further argue that although choosing mixed bundling over pure bundling can bring economic benefits, the magnitude of such benefits is often limited.

We next consider combining pay-per-use and subscription in the usage dimension, a strategy we term as *pay-per-use & subscription*. Since pay-per-use and subscription resemble component selling and pure bundling in the usage dimension, respectively, a combination of these two schemes resembles applying mixed bundling to the usage dimension. Under this new strategy, customers self-select in their usage plans, creating room for better price discrimination. We analyze this new strategy under component selling and pure bundling on variety. Our analysis suggests that mixed bundling on usage is often more profitable than mixed bundling on variety. We also find that pay-per-use & subscription is optimal in most cases, and that even in cases in which it is not optimal, it can perform reasonably well relative to the optimal strategy. This suggests some robustness of the pay-per-use & subscription scheme to various different settings.

Finally, we study nonlinear pricing by extending the pay-per-use scheme to allow prices that are contingent on a customer's past purchase. Noting the analytical challenge of the multi-dimensional mechanism design problem, we use a partial analysis of component selling and pure bundling to demonstrate the applicability of nonlinear pricing to settings with diminishing margins of consumption. We show that pure bundling on variety, which is unprofitable under linear pricing, can be a profitable strategy under nonlinear pricing.

We also study various extensions, including general valuation distributions, correlated valuations, asymmetric services, and heterogeneous diminishing rates of consumption, and show that our main results qualitatively carry through.

1.1 Related Literature

Our work bridges two streams of literature, each solving the pricing problem in one dimension of variety or usage. The price decision on variety is analyzed in a product bundling literature dating back to Adams et al. (1976). The core idea of this literature is that product bundling entails higher profitability by reducing customers' valuation dispersion. This idea is formalized by Fang and Norman (2006) and Ibragimov and Walden (2010) for a special class of (symmetric and log-concave) valuation distributions. Research in this literature also demonstrates the effects of other critical factors on the profitability of bundling, such as correlated valuations (Schmalensee, 1984; McAfee et al., 1989) and marginal cost (Bakos and Brynjolfsson, 1999; Wu et al., 2008). See Venkatesh and Kamakura (2003) for a comprehensive review of classic results on product bundling. There is a growing interest in applying product bundling to operational and marketing contexts. McCardle et al. (2007) studies the effect of demand uncertainty on a newsboy's bundling decision. Cao et al. (2015) studied a seller's bundling decision when one of the bundle components is subject to a capacity constraint. Banciu et al. (2010) studied how to bundle vertically differentiated products. Bhargava (2012) and Chakravarty et al. (2013) studied a retailer's bundling decision in a distribution channel. Wu et al. (2022) studied how to use product bundling to manage customer search in the face of valuation uncertainty. Recent empirical and experimental studies have also documented customers' behavioral issues in the presence of product bundling. Janiszewski and Cunha (2004) found a reference effect that appears in customers' perceptions of price discounts in bundles. Derdenger and Kumar (2013) reported a dynamic effect of product bundling on market segmentation outcomes. Chao and Derdenger (2013) presented evidence of installed base effects in two-sided markets and proposes mixed bundling as a device of price discrimination. Notably, most research in this literature assumes single-unit demands and this trivially precludes a meaningful analysis in the usage dimension.

The price decision on usage invites a literature on pay-per-use and subscription pricing. Randhawa and Kumar (2008) and Cachon and Feldman (2011) studied the comparison between these two schemes when customers' consumption of a service creates congestion externalities. Balasubramanian et al. (2015) and Ladas et al. (2022) studied this comparison in competitive settings. Focusing on information goods (for which customers' demands can be unbounded), Sundararajan (2004) proves the optimality of offering a combination of pay-per-use and subscriptions (similar to our pay-per-use & subscription scheme). The pay-per-use scheme is a special case of the more broad two-part tariffs studied by Masuda and Whang (2006) for markets with congestion externalities and by Png and Wang (2010) for markets with buyers' demand uncertainty. This literature does not consider how to sell multiple products or services; thus, all pricing variations in this literature are rooted exclusively in the usage dimension.

The only exceptions with an integral analysis of variety and usage are Armstrong (1996) and Armstrong and Vickers (2010). Armstrong (1996) studies a monopoly firm's optimal bundling decision under a special class of customers' utility functions (which are homogeneous of degree m in consumption profiles and thus fail to capture the diminishing margins of consumption). Armstrong and Vickers (2010) identified a set of equilibria for competing firms' multi-product price decisions. Our paper differs from Armstrong (1996) and Armstrong and Vickers (2010) mainly in two aspects. First, the pricing schemes we study in this article are motivated by real practice. Specifically, we study component selling, pure bundling, and mixed bundling in the variety dimension, and pay-per-use, subscriptions, and a combination of the two in the usage dimension. This endows our analysis and results with practical relevance. Second, unlike Armstrong (1996) which considers restricted utility functions and Armstrong and Vickers (2010) which assumes full market coverage of each product, our multi-unit demand model captures customers' diminishing margins of consumption while allowing the market coverage of each product to be endogenized by the firm's adopted pricing tactics.

2 The Model

We consider a monopoly firm (he) selling two different services (e.g., an amusement park that operates two contiguous parks[1]), each with a zero marginal cost.[2] Throughout the article, we interchangeably refer to "customers" and "users," and their "consumption" and "usage" as the same. A defining feature of our customers (she) is that they demand multiple units of each service. For example, visitors may patronize an amusement park multiple times throughout a year. We assume that customers have diminishing margins for the consumption of the same service. Specifically, a customer's marginal utility of consuming the n^{th} unit of service i for $i = 1, 2$, is $\theta_i - n$, where θ_i represents the customer's intrinsic valuation of that service and is a random draw from a probability density function (pdf) f_i with support on $[\underline{\theta}_i, \bar{\theta}_i]$.[3] For tractability we treat a customer's purchase and consumption quantities as continuous variables. This allows us to recover the commonly used quadratic utility functions for multi-unit consumption: a customer with intrinsic valuation θ_i receives an aggregate utility $\theta_i n - n^2/2$ by consuming n units of service i. Notice that the customer's intrinsic valuation θ_i also defines her *satiation point*, namely, the customer's highest consumption level of service i should it be offered for free. A customer will never consume a service more than her satiation point.

Our customer utility model has merits in three aspects. First, it provides a simple framework to capture customers' diminishing margins of consumption while affording tractability. Second, it facilitates a straightforward comparison to the classic product bundling literature on single-unit demands. This literature also assumes customers' valuations (θ_1, θ_2) as random draws from a common joint distribution, but specifies a customer's purchase behavior by comparing these valuations to a firm's posted prices. If a customer's valuation exceeds the price, then the customer will purchase a single unit of the product and consume it all. Our model deviates from this classic set-up by endogenizing customers' joint *variety* and *volume* decisions for their purchased products or services. This creates an important nuance under the pay-per-use scheme and leads to novel comparison results between component selling and pure bundling (see analysis in Section 3.2). Third, we establish the notion of satiation point using linear marginal utilities. This is consistent with a growing literature on multi-unit demands (e.g., Chun and Ovchinnikov, 2019; Agrawal and Bellos, 2017; Ladas et al., 2022 in the single-product setting, and Goić et al., 2011; Jerath and Zhang, 2010 in the multi-product setting).

Motivated by the common practice in the service industry, we focus on comparing pricing schemes that vary in two dimensions, *usage* and *variety*. Existing literature has solved the pricing problem in each of these dimensions, but it is not clear from prior research how to devise pricing strategies that involve both of them. To study this problem, we first fix pay-per-use in the usage dimension and examine whether to adopt bundling in the variety dimension. Specifically, under the pay-per-use scheme we study component selling that sells single access to each service, pure bundling that sells single access to both services, and mixed bundling that sells single access both to each service and to the service bundle. We focus on linear pricing irrespective of the (un)bundling decision on variety, that is, each user pays a fixed fee per use of a service or service bundle. Linear pricing has advantages in its simplicity and is a common subject of study in the pay-per-use literature (e.g., Chun and Ovchinnikov, 2019; Agrawal and Bellos, 2017; Ladas et al., 2022). However, we acknowledge that in reality firms may deviate from linear pricing and adopt more sophisticated nonlinear pricing, and we study nonlinear pricing in Section 7.

We next consider subscription pricing that grants subscribers unlimited access to a service. Subscription pricing essentially folds a customer's usage, irrespective of the amount, into a fixed fee. Similar to pay-per-use, under subscription pricing we study component selling that sells service-specific subscriptions, pure bundling that sells subscriptions of the service bundle, and mixed bundling that sells subscriptions both of each service and of the service bundle. Then, through a comparison between pay-per-use and subscriptions under different variety strategies, we provide insights into the optimal strategy that spans both usage and variety.

To draw insights, we follow the convention in the product bundling literature and assume that customers' intrinsic valuations of the two services are drawn independently from the same distribution (e.g., Fang and Norman, 2006; Ibragimov and Walden, 2010). To further simplify analysis, we focus on uniform valuations in our main analysis (e.g., Chun and Ovchinnikov, 2019; Ladas et al., 2022). We normalize the maximal valuation of each service to 1 (by applying an appropriate scaling when necessary) so that the valuations of each service are uniform over [0,1]. Noting the relative restrictiveness of uniform valuations, we also complement our analysis by discussing whether results derived under uniform valuations can extend to general valuation distributions; if not, what new insights can be obtained by considering non-uniform valuation distributions.

3 Pay-Per-Use

We start by analyzing pay-per-use that charges customers service fees on a per-use basis. Under linear pricing, customers' payments are proportional to the amount they patronize a service or service bundle. Following ideas in the product bundling literature, we study component selling, pure bundling, and mixed bundling with price variations in the variety dimension, and compare their profitability.

3.1 Component Selling Under Pay-Per-Use

We start by analyzing component selling. Let p_i denote the price charged per use of service i. Under component selling, each customer will consume a service as much she purchases it. A customer with valuation θ_i will continue purchasing service i, given her current purchase quantity n_i, if $\theta_i - n_i - p_i \geq 0$. Thus, the purchase (and consumption) quantities of a customer with valuations (θ_1, θ_2) are given by

$$n_1(\theta_1, \theta_2, p_1) = (\theta_1 - p_1)^+, \; n_2(\theta_1, \theta_2, p_2) = (\theta_2 - p_2)^+, \quad (1)$$

where $(\cdot)^+ := \max(\cdot, 0)$. Note that a customer's purchase quantity of service i will never exceed her satiation point θ_i, but can be possibly zero if her intrinsic valuation θ_i is lower than the pay-per-use price p_i.

Anticipating (1), the firm selects prices to maximize revenue,

$$\max_{p_i} \sum_{i=1,2} p_i \int n_i(\theta_1, \theta_2, p_i) f_i(\theta_i) d\theta_i.$$

The next result characterizes the optimal price under this pricing scheme, where we use subscript "CP" to denote component selling under pay-per-use.

PROPOSITION 1 (Component Selling Under Pay-Per-Use). *Suppose* $\theta_i \sim U(0,1)$, $i \in \{1,2\}$. *The optimal pay-per-use price under component selling is* $p_i^* = 1/3$ *and the optimal revenue is* $\Pi_{CP}^* = 4/27 \approx 0.148$.

The optimal pay-per-use price of each service is $1/3$ under component selling, which is lower than 1/2, the optimal component price in the single-unit demand model under uniform valuations. To explain this finding, note that customers' diminishing margins of consumption imply that they are unwilling to pay as much as they do as they continue purchasing and consuming the same service. Aware of this, the firm has to lower his pay-per-use price to stimulate more continued purchases.

3.2 Pure Bundling Under Pay-Per-Use

We next analyze pure bundling that sells single access to a service bundle.[4] Let p_B denote the bundle price. Notice that although paying price p_B grants access to both services, a paying customer is not obligated to consume them both. It is likely that some customers purchase the bundle with the intention of consuming only one service. Nevertheless, they have to pay p_B even though the bundle is only partially consumed.

Formally, with n_B denoting a customer's purchase quantity of the service bundle, the customer's consumption of service i is $\min\{\theta_i, n_B\}$, that is, it is constrained by both the customer's satiation point and her purchase quantity of the bundle. Then, a customer with valuations (θ_1, θ_2) will continue purchasing the bundle, given her current purchase quantity n_B, if

$$(\theta_1 - n_B)^+ + (\theta_2 - n_B)^+ - p_B \geq 0. \tag{2}$$

To explain (2), note that a customer will not consume a service more than her satiation point, so service i contributes a share of $(\theta_i - n_B)^+$ to the customer's marginal utility of purchasing the bundle. The customer will continue purchasing the bundle if the aggregate marginal utilities of consumption are greater than the bundle price; the customer will stop purchasing the bundle at quantity n_B as soon as (2) fails.

We next characterize a customer's purchase quantity of the bundle using (2). Let k denote the index of a customer's higher valuation between θ_1 and θ_2 (i.e., $k = 1$ if $\theta_1 \geq \theta_2$ and $k = 2$ otherwise) and $-k$ denote the index of the other valuation.

Lemma 1. *Under pure bundling and pay-per-use, a customer's purchase quantity of the bundle, $n_B(\theta_1, \theta_2, p_B)$, is given by*

$$n_B(\theta_1, \theta_2, p_B) = \begin{cases} \theta_k - p_B, & \text{if } p_B \leq \theta_k - \theta_{-k}, \\ \frac{\theta_1 + \theta_2 - p_B}{2}, & \text{if } \theta_k - \theta_{-k} \leq p_B \leq \theta_1 + \theta_2, \\ 0, & \text{otherwise}. \end{cases}$$

For illustrative purposes, in what follows, whenever we refer to a customer's purchase and consumption decisions, we follow the convention that the customer's valuations satisfy $\theta_1 \geq \theta_2$ so that the customer values service 1 more than service 2 (the case $\theta_2 > \theta_1$ can be analyzed analogously). When $p_B \leq \theta_1 - \theta_2$, consuming service 1 alone can generate a high utility, in which case, the customer purchases the bundle for an excessive amount above θ_2, the satiation point of service 2, and forgoes consuming service 2 while purchasing this amount. When $\theta_1 - \theta_2 \leq p_B \leq \theta_1 + \theta_2$, the customer consumes both services as she purchases the bundle, hitting neither of her satiation points. When $p_B > \theta_1 + \theta_2$, the bundle is prohibitively expensive and the customer does not purchase any amount of the bundle. A critical observation drawn from these three cases is that, for a customer with highly split valuations of two services ($\theta_1 - \theta_2 \geq p_B$), her purchase and consumption decisions for her less preferred service are not identical. This feature distinguishes pure bundling from component selling under the pay-per-use scheme, and is robust and can hold under any valuation distributions.

Anticipating customers' purchase behaviors under pure bundling as in Lemma 1, the firm selects the bundle price to maximize revenue,

$$\max_{p_B} \iint p_B n_B(\theta_1, \theta_2, p_B) f_1(\theta_1) f_2(\theta_2) d\theta_1 d\theta_2.$$

We characterize the optimal bundle price under pay-per-use in the next result, where we use subscript "BP" to denote pure bundling under pay-per-use.

Proposition 2 (Pure Bundling Under Pay-Per-Use). *Suppose $\theta_i \sim U(0,1)$, $i \in \{1,2\}$. Under pure bundling and pay-per-use, the optimal bundle price is $p_B^* = 1/2$ and the optimal revenue is $\Pi_{BP}^* = 9/64 \approx 0.141$. Hence, under pay-per-use, the firm's revenue is strictly lower under pure bundling than under component selling.*

According to Proposition 2, the firm charges a lower pay-per-use price, $1/4$, for each service under pure bundling (recall from Proposition 1 that this price is 1/3 under component selling). As we elaborate below, this is because, selling two services in a bundle reduces customers' incentives of purchasing their preferred service. This reduction in incentives is important to a majority of customers and it undermines the firm's pricing capabilities.

To see this more clearly, consider setting the bundle price $p_B = 2p$, where p is an arbitrary single-service price under component selling. We compare the total sales of the bundle under price $p_B = 2p$ with the sales of each service under component price p. Consider a representative customer with valuations $\theta_1 > \theta_2$. The total sales of the two services from this customer under component selling has a unified form of $(\theta_1 - p)^+ + (\theta_2 - p)^+$, but the sales of the bundle from this customer under pure bundling depends on the segment she belongs to. First, suppose the customer's valuations satisfy $\theta_1 + \theta_2 \leq 2p$. Then, by Lemma 1, this customer does not purchase any bundle. However, it is still possible that $\theta_1 > p$ which implies that the customer will purchase service 1 for a positive amount under component selling. Thus, pure bundling generates lower sales from this customer than component selling.

Next, consider a customer with $\theta_1 - \theta_2 < 2p < \theta_1 + \theta_2$. She purchases $n_B = (\theta_1 - p)/2 + (\theta_2 - p)/2$ units of the bundle, or equivalently, a total $(\theta_1 - p) + (\theta_2 - p)$ units of two services. When $\theta_2 < p$, this total demand is strictly lower than the total sales of the two services under component selling. This happens when service 2 has some moderate appeal, and the incentive of purchasing service 2 more than the customer's actual consumption is inhibited (in a positive way) by the high price under component selling. Now, under pure bundling, service 2 will still be purchased as part of the bundle, although the net utility of purchasing this service alone at price p is negative, that is, $(\theta_2 - n)^+ - p < 0$ for large n. However, the net utility of purchasing service 1, that is, the customer's preferred service, at price p, $(\theta_1 - n) - p$ (we remove the "+" sign as we assume $\theta_1 > \theta_2$), can be sufficiently positive so that the overall net utility of purchasing the bundle at price $p_B = 2p$ is positive. In this case, to continue purchasing the bundle, the utility of purchasing service 1 must compensate for the disutility of purchasing service 2. This compensation effect reduces the customer's incentive of purchasing service 1 in the first place. As a result, pure bundling generates lower sales from this customer than component selling.

The only segment that will potentially contribute more sales under pure bundling are those with $\theta_1 - \theta_2 > 2p$. These customers will forgo consuming service 2 for the excessive amount purchased above θ_2. This allows pure bundling to generate more sales from service 2 by forcing these customers to purchase their less preferred service more than their actual consumption. This is a dominant effect among those with valuations that satisfy $\theta_1 - 3p > (\theta_2 - p)^+$ (which happens with zero probability for $p \geq 1/3$). That is, extremely high valuations of service 1 will prompt a high-volume purchase of the bundle and generate favorably more sales of service 2. However, for customers with $\theta_1 - 3p \leq (\theta_2 - p)^+$, the aforementioned compensation effect remains dominant, so the sales from these customers are still lower under pure bundling than under component selling.

To summarize, under the pay-per-use scheme, pure bundling generates lower sales among a majority of customers except for those with $\theta_1 - 3p > (\theta_2 - p)^+$. This latter segment vanishes when p is set to be 1/3, the optimal pay-per-use price under component selling. Hence, the bundle sales under $p_B = 2p = 2/3$ (which is 16/81) are strictly lower than the sales of each service under component selling with price $p = 1/3$ (which is 18/81).

Our finding is in contrast to classic single-unit demand models in the product bundling literature which suggest that by setting the bundle price $p_B = 2p$ (where p is the optimal price under component selling), the sales of each product tend to be higher under pure bundling due to reduced valuation dispersion (e.g., Fang and Norman, 2006; see also our discussion in Section 4.2). In these models, pure bundling is often a more profitable strategy and the firm *actively* lowers the bundle price in seek of higher sales. In contrast, in our multi-unit demand model, pure bundling falls short of component selling under $p_B = 2p$, and the firm has to unwillingly lower the bundle price to stimulate sales already pushed to a low level due to the compensation effect. As a result, component selling can outperform pure bundling (roughly 5.4% more in revenue under uniform valuations). In other words, the compensation effect engenders an inefficiency in "bundling variety" under the pay-per-use scheme. This is a robust result that extends to non-uniform valuations (see Section 6.1). This also motivates us to consider more refined pricing schemes to mitigate this inefficiency.

3.3 Mixed Bundling Under Pay-Per-Use

In the previous analysis, we focused on pure bundling on variety. That is, all services are lumped together into a single package. In this section we study mixed bundling under pay-per-use: the firm sells single access both to each service and to the service bundle. As we will show later, the use of mixed bundling can eliminate the adverse compensation effect under pure bundling.

Mixed bundling is known as the most profitable pricing strategy in the product bundling literature on single-unit demands (McAfee et al., 1989).[5] Despite its well-known economic benefits, mixed bundling is also known as being analytically intractable even in very simple settings due to the complicated segmentations it creates (Venkatesh and Kamakura, 2003). Integrating mixed bundling with pay-per-use in our setting generates seemingly more involved customers' purchase behaviors with variations in both usage and variety. Thus, the analysis in the product bundling literature does not directly apply.

Formally, let p_B denote the price per use of a service bundle and p denote the price per use of a service (by symmetry, we assume this price is identical across two services). To rule out triviality, we impose $p < p_B < 2p$. Let n_B denote a customer's purchase quantity of the service bundle and n_i denote the customer's purchase quantity of service i.

The following lemma is useful in describing a customer's purchase decisions under mixed bundling. To state our result, recall that k denotes the index of a customer's higher valuation between θ_1 and θ_2, and $-k$ denotes the index of the other valuation.

LEMMA 2. *Under mixed bundling and pay-per-use, if a customer purchases n_B units of the bundle, she will consume all n_B units of the bundle. Moreover, the customer will not purchase any additional amount of her less preferred service, that is, $n_{-k} = 0$.*

Lemma 2 suggests that under mixed bundling, each customer will first purchase a base amount of the bundle, consume it all, and then purchase an additional amount of her preferred service. Thus, unlike pure bundling, under mixed bundling customers always fully consume both bundle components. This implies that $n_k = (\theta_k - n_B - p)^+$ and $n_{-k} = 0$. Next, to derive n_B, consider a customer with valuations (θ_1, θ_2) who has purchased n_B units of the bundle. The customer will continue purchasing the bundle if

$$(\theta_1 - n_B)^+ + (\theta_2 - n_B)^+ - p_B \geq \max\{(\theta_1 - n_B)^+ - p, (\theta_2 - n_B)^+ - p, 0\}. \quad (3)$$

The left hand side of (3) represents the marginal utility of purchasing the bundle, which is the aggregate utility from optimally consuming two services, $(\theta_1 - p_B)^+$ and $(\theta_2 - p_B)^+$, net the bundle price p_B. To further purchase the bundle, this net utility must be greater than the utility of purchasing only one service as well as the utility of forgoing any purchase. Note that the satiation point appears in (3) by applying "+" signs to both sides. Solving (3) gives each customer's purchase decisions presented below.

LEMMA 3. *Under mixed bundling and pay-per-use, consider a customer with valuations* (θ_1, θ_2).

1. *If* $p_B \geq \theta_1 + \theta_2$, *then the customer purchases* $(\theta_k - p)^+$ *units of service k.*
2. *If* $\theta_k - \theta_{-k} \leq p_B \leq \theta_1 + \theta_2$, *then the customer purchases* $n_B = \min\{(\theta_1 + \theta_2 - p_B)/2, (\theta_{-k} - p_B + p)^+\}$ *units of the bundle and* $n_k = (\theta_k - n_B - p)^+$ *units of service k.*
3. *If* $p_B \leq \theta_k - \theta_{-k}$, *then the customer purchases* $n_B = (\theta_{-k} - p_B + p)^+$ *units of the bundle and* $n_k = (\theta_k - n_B - p)^+$ *units of service k.*

To understand Lemma 3, consider again a customer with valuations $\theta_1 > \theta_2$. We decompose (3) into two separate constraints: an incentive compatibility (IC) constraint,

$$(\theta_1 - n_B)^+ + (\theta_2 - n_B)^+ - p_B \geq \max\{(\theta_1 - n_B)^+ - p, (\theta_2 - n_B)^+ - p\},$$

and an individual rationality (IR) constraint same as (2). Since we assume $\theta_1 > \theta_2$, we can simplify the IC constraint to $\theta_2 - n_B \geq p_B - p$, which implies $n_B \leq (\theta_2 - p_B + p)^+$. Combining this with Lemma 1, we obtain Lemma 3.

Lemma 3 demonstrates a rich pattern of customers' purchase behaviors under mixed bundling. Unlike those under pure bundling which are fully specified by an IR constraint, customers' purchase behaviors under mixed bundling are also determined by an IC constraint that specifies that a customer will not continue purchasing the bundle unless the marginal utility of consuming her less preferred service is not too small; otherwise, the customer will be better off by purchasing her preferred service standalone. In this way, mixed bundling allows customers to flexibly purchase their preferred service without being forcefully tied to their less preferred service. Thus, mixed bundling eliminates the inconsistency between customers' purchase and consumption decisions for their less preferred service. This effectively allays the compensation effect and hints at a potential revenue improvement under mixed bundling.

Anticipating customers' purchase behaviors under mixed bundling as in Lemma 3, the firm selects prices p_B and p to maximize revenue,

$$\max_{p_B, p} \iint [p_B n_B(\theta_1, \theta_2, p_B, p) + p n_1(\theta_1, \theta_2, p_B, p) + p n_2(\theta_1, \theta_2, p_B, p)] f_1(\theta_1) f_2(\theta_2) d\theta_1 d\theta_2.$$

The next result shows that under pay-per-use, mixed bundling is superior to both pure bundling and component selling.

PROPOSITION 3 (Mixed Bundling Under Pay-Per-Use). *Suppose* $\theta_i \sim U(0,1)$, $i \in \{1,2\}$. *Then, under pay-per-use, mixed bundling generates a strictly higher revenue than both component selling and pure bundling.*

As said, mixed bundling is known as being analytically intractable even in simple settings of single-unit demands (Venkatesh and Kamakura, 2003). The multi-unit-demand assumption in our model further complicates this analysis by endogenizing customers' purchase decisions in both variety and usage (cf. Lemma 3), so we are unable to derive in closed forms the optimal prices under mixed bundling. In the proof of Proposition 3, we propose a feasible solution of mixed bundling, as motivated by the optimal price under component selling, and show that this solution achieves a revenue strictly higher than the optimal revenue under component selling. We numerically find that the optimal revenue under mixed bundling is 0.152 (roughly 2.6% more than that under component selling) achieved by setting $p^* = 0.380$ and $p_B^* = 0.602$. Finally, although the dominance of mixed bundling over pure bundling and component selling is established under uniform valuations, such dominance can in fact extend to general valuation distributions (see Section 6.1).

4 Subscription Pricing

In this section, we study subscription pricing, another commonly used usage-based pricing scheme in the service industry. Different from the pay-per-use scheme that charges on-demand usage, subscription pricing takes a "bundling" approach in the usage dimension: it sets up a fixed fee that grants paying subscribers unlimited access to a service. Using similar ideas in Section 3, we study component selling, pure bundling, and mixed bundling in the variety dimension by fixing subscription pricing in the usage dimension.

Under subscription pricing, a customer with intrinsic valuation θ_i of service i, upon subscription, will consume an amount equal to her satiation point θ_i of this service and receive a gross utility $\theta_i^2/2$ from consumption. Thus, at an aggregate level, customers' valuations of subscribing to service i are distributed according to $V_i := \theta_i^2/2$, with θ_i being the intrinsic valuations of service i. In this way, it suffices to consider how to bundle different subscriptions, with valuations V_i of each subscription distributed as $V_i = \theta_i^2/2$. Treating each subscription as a single "product," some results in this section follow directly from the product bundling literature.

4.1 Component Selling Under Subscription

We first consider component selling that offers service-specific subscriptions. Let p_i denote the subscription fee of service i. Then, a customer with intrinsic valuation θ_i of service i will buy subscription i if $V_i = \theta_i^2/2 \geq p_i$. Anticipating this, the firm selects subscription fee p_i to maximize revenue,

$$\max_{p_i} \sum_{i=1,2} p_i \mathbb{P}\left\{\theta_i^2/2 \geq p_i\right\}.$$

The following result characterizes the optimal component price under subscriptions, where we use subscript "CS" to denote component selling under subscriptions.

PROPOSITION 4 (Component Selling Under Subscription). *Suppose $\theta_i \sim U(0,1)$, $i \in \{1,2\}$. Under component selling and subscriptions, the optimal subscription fee is $p_i^* = 2/9$ and the optimal revenue is $\Pi_{CS}^* = 4/27$. Hence, under uniform valuations, component selling achieves the same revenue under pay-per-use and under subscriptions.*

Comparing component selling across the pay-per-use and subscription schemes, we find that somewhat surprisingly, the resulting revenues are identical under uniform valuations. However, we mention that this revenue equivalence only applies to uniform valuations and does not extend to other valuation distributions. In general, the comparison between pay-per-use and subscriptions under component selling is highly mixed and depends on the entire valuation distribution. In fact, in Section 6.1, we provide cases of valuation distributions in which either usage-based pricing strategy can dominate the other. Specifically, when customers' valuations are distributed according to Beta(1,3) and Beta(2,5), pay-per-use will dominate subscriptions under component selling. Note that these two distributions both have *decreasing right tails* (see Figure 1 in Appendix A in the E-Companion), implying that only a small mass of customers have high valuations of each service. Since a customer's intrinsic valuation also defines her satiation point, these customers have high demands for each service too. In this sense, our finding echoes with Varian (2000) in that pay-per-use is often more profitable to sell to markets with low individual demands. This is because, pay-per-use can generate sales from low-valuation customers even though they do not purchase much, whereas subscriptions can only be sold exclusively to customers with sufficiently high valuations. Thus, when low-valuation customers constitute a significant portion of the market, it is better to choose pay-per-use over subscriptions. In contrast, when customers' valuations are distributed according to Beta(0.5,0.5) and Beta(3,1), both having *increasing right tails*, subscriptions can be more profitable because it can effectively extract surplus from high-valuation customers.

4.2 Pure Bundling Under Subscription

We next analyze pure bundling under subscriptions that grants subscribers unlimited access to both services. Examples include the "2-Park Annual Pass" of Universal Orlando.[6] Let p_B denote the subscription fee of the service bundle. A customer with intrinsic valuations (θ_1, θ_2) will buy the subscription if $V_1 + V_2 = (\theta_1^2 + \theta_2^2)/2 \geq p_B$. Anticipating this, the firm selects p_B to maximize revenue,

$$\max_{p_B} p_B \mathbb{P}\left\{\frac{\theta_1^2 + \theta_2^2}{2} \geq p_B\right\}.$$

As before, we use subscript "BS" to denote pure bundling under subscriptions for the result stated below.

PROPOSITION 5 (Pure Bundling Under Subscription). *Suppose $\theta_i \sim U(0,1)$, $i \in \{1,2\}$. Under pure bundling and subscriptions, the optimal subscription fee is $1/\pi \approx 0.318$ and the optimal revenue is $\Pi_{BS}^* = 1/(2\pi) \approx 0.159$. Hence, under subscriptions, the firm's revenue is strictly higher under pure bundling than under component selling.*

The optimal prices and revenues are explicitly stated in Proposition 5 for uniform valuations, but they in general cannot be derived in closed forms for non-uniform valuations. However, the dominance of pure bundling over component selling under subscriptions can hold rather generally. It is also worth noting that such dominance can often be significant in magnitude (such dominance corresponds to a 7.3% relative difference under uniform valuations).

To summarize, under uniform valuations, pure bundling under subscriptions dominates both component selling under subscriptions (Proposition 5) and under pay-per-use (Proposition 1), and they all dominate pure bundling under pay-per-use (Proposition 2). This implies that the compensation effect that exists under pure bundling and pay-per-use fully disappears under all subscription schemes. This occurs because, under subscriptions, customers have unrestricted access to the services, and this simplifies the usage dimension of the pricing problem faced by the firm, thereby inducing customers to consume quantities that maximize their gross utility, fully reaching their satiation points (θ_i). This principle holds true under both component selling and pure bundling. Consequently, customers' valuations of each service (taking account of the usage) points to a singular metric: the maximum gross utility derived from service consumption ($V_i = \theta_i^2/2$). As a result, the compensation effect, wherein customers are compelled to purchase an entire bundle merely to access a preferred service, vanishes under subscriptions.

4.3 Mixed Bundling Under Subscription

We next consider mixed bundling under subscriptions. In this case, customers can subscribe to either a service bundle or a single service. Let p_B and p denote the subscription fees of the service bundle and each single service, respectively (by symmetry we assume the single-service fee p is identical across two services). We write the firm's revenue under this pricing scheme as follows:

$$\max_{p_B,p}\ p_B\mathbb{P}\left\{\frac{\theta_1^2+\theta_2^2}{2}\ge p_B, \frac{\theta_1^2}{2}\ge p_B-p, \frac{\theta_2^2}{2}\ge p_B-p\right\}$$
$$+p\mathbb{P}\left\{\frac{\theta_1^2}{2}\ge p, \frac{\theta_2^2}{2}< p_B-p\right\}$$
$$+p\mathbb{P}\left\{\frac{\theta_2^2}{2}\ge p, \frac{\theta_1^2}{2}< p_B-p\right\}.$$

Although we are unable to derive in closed forms the optimal prices under mixed bundling, McAfee et al. (1989), Corollary 1 postulates that mixed bundling under subscriptions always *strictly* dominates component selling under subscriptions.

PROPOSITION 6 (Mixed Bundling Under Subscription). *Under subscriptions, mixed bundling generates a strictly higher revenue than component selling for all valuation distributions.*

Focusing on uniform valuations, we numerically find that the optimal subscription fees under mixed bundling are $p_B^* = 0.432$ and $p^* = 0.280$, generating a total revenue of 0.160. This revenue is only slightly higher than that under pure bundling and subscriptions (roughly 0.6% more). This echoes with a less noticed observation in the product bundling literature: despite the well-known economic benefits of mixed bundling, the magnitude of such benefits, especially over pure bundling, is often limited.[7] So, the firm must tradeoff the limited economic benefits of a more involved "mixed" strategy on variety against the cost of managing multiple prices. In general, we advocate considering both pure bundling and mixed bundling to sell subscriptions. We recommend mixed bundling if managing multiple prices is costless, and recommend pure bundling if there is a preference to adhere to simplicity and implement as few prices as possible. Indeed, in practice, Disney, for example, offers multiple subscription options to manage its streaming services: a customer can subscribe to one of Hulu, Disney+, and ESPN+, or to a Hulu/Disney+ bundle, or to a Hulu/Disney+/ESPN+ bundle. This resembles the scheme of mixed bundling under subscriptions, driven possibly by the fact that the management of pricing for digital subscriptions is often near-costless.

5 Bundling Usage, Variety, or Both?

In the previous analysis, we chose to vary the price decision on variety while fixing the price decision on usage (under either pay-per-use or subscriptions). In this section, we first analyze a combination of pay-per-use and subscriptions in the usage dimension, which resembles applying *mixed bundling* to this dimension. We then make a thorough comparison between all pricing strategies that involve both usage and variety.

5.1 Pay-Per-Use & Subscription

In previous sections, we analyzed pay-per-use and subscriptions, each resembling component selling and pure bundling in the usage dimension. To complete our analysis, in this section, we consider combining these two strategies, that is, we apply mixed bundling to the usage dimension. This strategy resembles the practice of amusement parks that offer a combination of annual and daily passes. We term this new strategy as *pay-per-use & subscription*.[8] To simplify analysis, we fix the price decision on variety to be either component selling or pure bundling.[9]

Component Selling Under Pay-Per-Use & Subscription. Under component selling on variety, let p_U and p_S denote the pay-per-use price and subscription fee of each service, respectively (by symmetry we assume these prices are identical across two services). Consider a customer with intrinsic valuation θ_i for service i. She has three purchase options regarding service i.

1. If she chooses to purchase service i under pay-per-use, she will purchase $(\theta_i - p_U)^+$ units of the service with a payoff
$$U_U(\theta_i) = \int_0^{(\theta_i-p_U)^+}(\theta_i - p_U - n)dn = \begin{cases}(\theta_i-p_U)^2/2, & \text{if } \theta_i \ge p_U,\\ 0, & \text{otherwise.}\end{cases}$$
2. If she chooses to purchase the subscription of service i, her payoff is $U_S(\theta_i) = \theta_i^2/2 - p_S$.
3. If she forgoes any purchase, she receives zero payoff $U_0 = 0$.

Let $\mathcal{S} = \{\theta : U_S(\theta) \ge \max\{U_U(\theta), 0\}\}$ and $\mathcal{U} = \{\theta : U_U(\theta) > \max\{U_S(\theta), 0\}\}$ be the customer segments who will purchase service i under subscriptions and under pay-per-use, respectively. Since these segments are mutually exclusive, we can write the firm's total revenue as

$$\max_{p_U,p_S}\quad 2\left[p_S\int_{\mathcal{S}} f_i(\theta_i)d\theta_i + p_U\int_{\mathcal{U}}(\theta_i - p_U)f_i(\theta_i)d\theta_i\right].$$

We solve the above problem and present the optimal prices in the following result. We use subscript "CPS" to denote component selling under pay-per-use & subscription.

PROPOSITION 7. *Suppose $\theta_i \sim U(0,1)$, $i \in \{1,2\}$. Under component selling and pay-per-use & subscription, the optimal pay-per-use price is $p_U^* = 2/5$, subscription fee is $p_S^* = 6/25$, and optimal revenue is $\Pi_{CPS}^* = 4/25 = 0.160$.*

Fixing component selling on variety, both the optimal pay-per-use price and subscription fee under the pay-per-use & subscription scheme are higher than their counterparts when only one of pay-per-use and subscriptions is used in the

Table 2. Revenue comparison between pay-per-use, subscriptions, and pay-per-use & subscription: uniform valuations.

	Pay-Per-Use			Subscription			Pay-Per-Use & Subscription	
Valuation distribution	CS	PB	MB	CS	PB	MB	CS	PB
Uniform	0.148	−5.1%	+2.5%	+0.0%	+7.4%	+8.0%	+8.0%	+**10.9**%

CS = component selling; PB = pure bundling; MB = mixed bundling.

usage dimension. This is reminiscent of the product bundling literature (on variety) which suggests that mixed bundling allows firms to raise both component and bundle prices. Thus, although the pay-per-use & subscription scheme resembles mixed bundling on usage, it allows the firm to raise both the "component" (corresponding to pay-per-use) and "bundle" (corresponding to subscription) prices in this dimension in a similar fashion to mixed bundling on variety.

According to Proposition 7, component selling under the pay-per-use & subscription scheme generates a revenue of 0.160 under uniform valuations, so it outperforms component selling under both pay-per-use and subscriptions (roughly 8.0% more). To understand how the pay-per-use & subscription scheme works, note that with pay-per-use & subscription, the firm selectively sells subscriptions to high-valuation customers and sells pay-per-use to low-valuation customers. Thus, this scheme extracts the respective advantages of pay-per-use and subscriptions, effectively creating a more efficient market segmentation.

Pure Bundling Under Pay-Per-Use & Subscription. Under pure bundling on variety, customers must purchase two services in bundles under either pay-per-use or subscriptions. Let p_U and p_S denote the pay-per-use price and subscription fee of the service bundle. Consider a customer with intrinsic valuations (θ_1, θ_2). She has three purchase options regarding the bundle.

1. If she chooses to purchase the bundle under pay-per-use, her purchase quantity of the bundle $n_B(\theta_1, \theta_2)$ is given in Lemma 1. Her payoff is

$$U_U(\theta_1, \theta_2) = \int_0^{n_B(\theta_1,\theta_2)} \left[(\theta_1 - n)^+ + (\theta_2 - n)^+ - p_U\right] dn.$$

2. If she chooses to subscribe to the bundle, her payoff is $U_S(\theta_1, \theta_2) = (\theta_1^2 + \theta_2^2)/2 - p_S$.
3. If she forgoes any purchase, she receives zero payoff $U_0 = 0$.

Let $\mathcal{S} = \{(\theta_1, \theta_2) : U_S(\theta_1, \theta_2) \geq \max\{U_U(\theta_1, \theta_2), 0\}\}$ and $\mathcal{U} = \{(\theta_1, \theta_2) : U_U(\theta_1, \theta_2) > \max\{U_S(\theta_1, \theta_2), 0\}\}$ be the customer segments who will purchase the bundle under subscriptions and pay-per-use, respectively. Using this segmentation, we can write the firm's revenue as

$$\max_{p_U, p_S} p_S \int_{\mathcal{S}} f_{1,2}(\theta_1, \theta_2) d\theta_1 d\theta_2 + p_U \int_{\mathcal{U}} n_B(\theta_1, \theta_2) f_{1,2}(\theta_1, \theta_2) d\theta_1 d\theta_2, \quad (4)$$

where $f_{1,2}$ denotes the joint pdf of (θ_1, θ_2).

We numerically solve (4) and find that the firm generates a revenue of 0.164 under uniform valuations, so pure bundling under pay-per-use & subscription is most profitable among all pricing schemes considered so far. Notice also that the preceding component selling under the pay-per-use & subscription scheme can perform reasonably well too. This suggests that mixed bundling on usage is in general useful to generate revenues. This is a robust result that can hold under many valuation distributions (see Section 6.1).

Table 2 summarizes the revenues of all pricing schemes under uniform valuations, where we use "CS," "PB," and "MB" to denote component selling, pure bundling, and mixed bundling, respectively. For an easy comparison, we report all revenues in relative values to that under component selling and pay-per-use, and we highlight the optimal pricing strategy in bold form.

6 Extensions

In this section, we consider various extensions to our main model. Specifically, we consider non-uniform valuation distributions, correlated service valuations, asymmetric services with vertical differentiation, and heterogeneous diminishing rates of consumption in these extensions. In each extension, we relax one assumption in the main model while fixing all other assumptions.

6.1 Non-Uniform Valuation Distributions

In this section, we examine the robustness of our comparison results between different pricing schemes established under uniform valuations. Specifically, we replicate our previous analysis of uniform valuations under various beta distributions. We select this class of distributions because they are able to capture a wide variety of distributional patterns that are representative of most realistic settings. Note that Beta(1,1) corresponds to the uniform distribution in our main model. In addition, we consider beta distributions with probability density functions that have a U-shaped (Beta (0.5,0.5)) and an inverse U-shaped (Beta(2,2), and Beta(2,5)), and that are

Table 3. Revenue comparison under pay-per-use: beta distributions.

Beta distribution	Pay-per-use Component selling	Pay-per-use Pure bundling	Pay-per-use Mixed bundling	Single-unit demand Component selling	Single-unit demand Pure bundling	Single-unit demand Mixed bundling
Beta(2,2)	0.134	−3.0%	+3.3%	0.520	+11.9%	+12.1%
Beta(2,5)	0.045	−0.8%	+4.5%	0.268	+14.0%	+14.0%
Beta(0.5,0.5)	0.170	−7.1%	+1.4%	0.525	+ 2.3%	+ 6.2%
Beta(1,3)	0.041	−0.8%	+3.8%	0.211	+12.5%	+12.5%
Beta(3,1)	0.285	−1.3%	+1.7%	0.945	+ 8.4%	+ 9.5%

Table 4. Revenue comparison under subscription pricing: beta distributions.

Beta distribution	Component selling under subscriptions	Pure bundling under subscriptions	Mixed bundling under subscriptions
Beta(2,2)	0.129	+11.9%	+11.9%
Beta(2,5)	0.039	+13.5%	+13.5%
Beta(0.5,0.5)	0.189	+ 0.9%	+ 4.5%
Beta(1,3)	0.035	+10.0%	+10.0%
Beta(3,1)	0.326	+ 9.0%	+10.2%

Table 5. Revenue comparison between pay-per-use, subscriptions, and pay-per-use & subscription.

	Pay-per-use			Subscription			Pay-per-use & subscription	
Beta distribution	CS	PB	MB	CS	PB	MB	CS	PB
Beta(1,1)	0.148	−5.1%	+2.5%	+ 0.0%	+ 7.4%	+ 8.0%	+ 8.0%	+**10.9**%
Beta(2,2)	0.134	−3.0%	+3.3%	− 3.9%	+ 7.6%	+ 7.6%	+ 4.9%	+**11.5**%
Beta(2,5)	0.045	−0.8%	+4.5%	−13.8%	− 2.1%	− 2.1%	+ 0.6%	+**5.0**%
Beta(0.5,0.5)	0.170	−7.1%	+1.4%	+11.2%	+12.2%	+16.2%	+**16.3**%	+14.1%
Beta(1,3)	0.041	−0.8%	+**3.8**%	−15.6%	− 7.2%	− 7.2%	+ 0.7%	+ 2.4%
Beta(3,1)	0.285	−1.3%	+1.7%	+14.2%	+24.4%	+**25.8**%	+17.7%	+25.6%

CS = component selling; PB = pure bundling; MB = mixed bundling.

monotone increasing (Beta(1,3)) and monotone decreasing (Beta(3,1)), respectively. We plot the probability density functions of these beta distributions in Figure 1 in Appendix A in the E-Companion.

Pay-Per-Use. Table 3 reports the comparison results of different pay-per-use schemes under various beta distributions ("pay-per-use" column), where we also contrast these results with those in the classic single-unit demand models ("Single-Unit Demand" column). Observe that, for all beta distributions considered, pure bundling is dominated by component selling, and they both are dominated by mixed bundling under the pay-per-use scheme. This suggests a robust compensation effect that undermines the profitability of pure bundling under pay-per-use as well as a robust economic benefit of mixed bundling in the variety dimension.

Subscriptions. Table 4 presents the results of subscription pricing under various beta distributions. These results are largely similar to those under uniform valuations. Specifically, under subscription pricing, in all cases except for Beta(0.5,0.5), there can be significant benefits from switching from component selling to pure bundling, but further switching to mixed bundling can only bring very limited gains. In fact, we numerically find that in certain cases (such as Beta(2,5)), the optimal $p_B^* = p^*$ under mixed bundling, implying that *mixed bundling effectively reduces to pure bundling*.

Pay-Per-Use & Subscription. We next consider pricing strategies that span both variety and usage. Table 5 reports the results under different beta distributions, including the uniform distribution beta(1,1). We use "CS," "PB," and "MB" to denote component selling, pure bundling, and mixed bundling, respectively. All revenues are reported in relative values to those under component selling and pay-per-use. We highlight in bold forms the optimal strategy under each valuation distribution.

We make three observations. First, mixed bundling on usage (i.e., pay-per-use & subscription) is generally more profitable than mixed bundling on variety. In many cases, the best strategy is obtained under pay-per-use & subscription (in conjunction with pure bundling on variety in most cases), and

Table 6. Revenue comparison between pay-per-use, subscriptions, and pay-per-use & subscription: correlated valuations.

	Pay-per-use			Subscription			Pay-per-use & Subscription	
Correlation	CS	PB	MB	CS	PB	MB	CS	PB
0.6	0.148	−3.4%	+0.8%	+0.0%	+ 0.6%	+ 2.2%	+**8.0**%	+ 5.8%
0.3	0.148	−4.7%	+1.6%	+0.0%	+ 3.6%	+ 4.7%	+**8.0**%	+ 7.8%
0	0.148	−5.1%	+2.5%	+0.0%	+ 7.4%	+ 8.0%	+8.0%	+**10.9**%
−0.3	0.148	−1.8%	+4.2%	+0.0%	+22.4%	+22.4%	+8.0%	+**22.4**%
−0.6	0.148	+1.7%	+6.4%	+0.0%	+42.2%	+42.2%	+8.0%	+**42.2**%

CS = component selling; PB = pure bundling; MB = mixed bundling.

even in cases in which pay-per-use & subscription is not optimal (Beta(1,3) and Beta(3,1)), it can perform reasonably well. So, the pay-per-use & subscription scheme is robust to different valuation distributions. This partially explains why most amusement parks choose to offer a combination of annual and daily passes.

Second, fixing pay-per-use or subscriptions in usage, mixed bundling on variety is more profitable than both component selling and pure bundling. Thus, mixed bundling is recommended if only one of the pay-per-use or subscription schemes is used. However, whether one should use pay-per-use or subscriptions to complement mixed bundling depends on the tail distributions of customers' valuations (see also our discussion in Section 4.1). In some cases (Beta(2,5) and Beta(1,3)), pay-per-use is better than subscriptions, whereas the reversal is true in other cases.

Third, the comparison between pure bundling and component selling on variety depends on the price decision on usage: pure bundling is always recommended over component selling under subscriptions but not under pay-per-use, and continues to be recommended in most cases under pay-per-use & subscription.

6.2 Correlated Valuations

In reality customers often have correlated valuations for two related services. In the classic single-unit demand models, McAfee et al. (1989) showed that a negative correlation between product valuations can further strengthen the economic benefits of pure bundling. Schmalensee (1984) shows that such a negative correlation is in fact not necessary to guarantee the success of pure bundling; pure bundling can also outperform separate selling under a positive correlation. In this section, we explore how correlated valuations will affect our results under multi-unit demands. The analysis of general correlations is often not tractable; see Wu et al. (2019) for a discussion on the analytical challenge of studying general correlations. To circumvent this challenge, the existing research typically focuses on specific correlation structures for an insightful analysis; for example, Schmalensee (1984); Armstrong and Vickers (2010), and Wu et al. (2022). We follow this convention and analyze a special class of correlations also considered by Armstrong and Vickers (2010) and Wu et al. (2022).

Following the base model, we assume that the marginal distributions of customers' valuations θ_1 and θ_2 are both uniform over [0,1]. Under a positive correlation, we assume that for any $t \in [0, 1]$, given a customer's intrinsic valuation for product i, $\theta_i = t$, the customer's valuation for the other product, $\theta_{3-i} = t$ with probability $\kappa \in [0, 1]$, and θ_{3-i} is uniformly distributed over [0,1] independently of θ_i with probability $1 - \kappa$. So, at an aggregate level, a κ fraction of customers have the same intrinsic valuations for both products and the remaining $1 - \kappa$ fraction of customers have independent intrinsic valuations. One can show that the joint distribution of θ_1 and θ_2 under this correlation is well-defined. Moreover, κ represents the correlation coefficient of θ_1 and θ_2 and thus measures the strength of correlation. Similarly, under a negative correlation, we assume that $\theta_1 = 1 - \theta_2$ with probability κ and that θ_1 and θ_2 are independent with probability $1 - \kappa$. The resulting correlation coefficient is $-\kappa$.

Under these correlations, we compute the optimal revenues under each pricing scheme and report our results in Table 6. We find that the comparison between separate selling and pure bundling under pay-per-use can be possibly reversed under negative correlations: this is the case when there exists a strong negative correlation (with correlation < -0.46; see Appendix B in the in the E-Companion for more analytical results developed for this comparison). Despite this important ramification, we confirm that mixed bundling on usage continues to be more profitable than mixed bundling on variety, especially in the case of positive correlations. However, under negative correlations, the gains from jointly using pay-per-use and subscriptions can be very limited; thus, one can simply use pure bundling and subscriptions to achieve almost all the economic benefits.

6.3 Asymmetric Services

Our main model focuses on two symmetric services with identical valuation distributions. We now relax this assumption and consider two asymmetric services with vertical differentiation. We assume that customers' intrinsic valuations of two services are distributed as θ_1 and $\delta\theta_2$, where $\delta < 1$ and θ_1, θ_2 have the same distribution. So, relative to service 1, service 2 is an *inferior* service with lower average valuations. Despite this vertical differentiation, customers' purchase behaviors mimic those in the main model and are fully determined by their

Table 7. Revenue comparison between pay-per-use, subscriptions, and pay-per-use & subscription: asymmetric services, $\delta = 0.6$.

	Pay-per-use			Subscription			Pay-per-use & subscription	
Beta distribution	CS	PB	MB	CS	PB	MB	CS	PB
Beta(1,1)	0.101	−4.7%	+0.8%	+ 0.0%	− 2.8%	+ 6.0%	+ **8.0**%	+ 5.7%
Beta(2,2)	0.091	−3.2%	+2.0%	− 3.9%	+ 0.6%	+ 4.0%	+ 4.9%	+ **7.0**%
Beta(2,5)	0.031	−0.4%	+**3.0**%	−13.8%	− 7.4%	− 6.4%	+ 0.6%	+ 1.9%
Beta(0.5,0.5)	0.116	−7.5%	+0.8%	+11.2%	+12.8%	+15.3%	+**16.3**%	+ 7.8%
Beta(1,3)	0.028	−0.7%	+**1.9**%	−15.6%	−12.9%	−10.5%	+ 0.7%	+ 0.3%
Beta(3,1)	0.194	−6.0%	+1.1%	+14.2%	+19.0%	+**22.9**%	+17.7%	+22.0%

CS = component selling; PB = pure bundling; MB = mixed bundling.

Table 8. Revenue comparison between pay-per-use, subscriptions, and pay-per-use & subscription: heterogeneous diminishing rates, $\lambda = 0.5, \gamma = 1.5$.

	Pay-per-use			Subscription			Pay-per-use & subscription	
Beta distribution	CS	PB	MB	CS	PB	MB	CS	PB
Beta(1,1)	0.123	−5.1%	+2.5%	− 3.0%	+ 3.1%	+ 3.9%	+ 4.3%	+ **6.1**%
Beta(2,2)	0.112	−3.0%	+3.3%	− 6.5%	+ 3.5%	+ 3.5%	+ 2.8%	+ **7.6**%
Beta(2,5)	0.038	−0.8%	+**4.5**%	−15.4%	− 4.7%	− 4.7%	+ 0.4%	+ 3.6%
Beta(0.5,0.5)	0.142	−7.1%	+1.4%	+ 6.4%	+ 7.7%	+**10.3**%	+ 9.6%	+ 6.7%
Beta(1,3)	0.034	−0.8%	+**3.8**%	−17.0%	− 9.1%	− 9.1%	+ 0.4%	+ 1.5%
Beta(3,1)	0.238	−1.3%	+1.7%	+ 8.5%	+15.4%	+**16.8**%	+11.3%	+16.5%

CS = component selling; PB = pure bundling; MB = mixed bundling.

respective valuations. However, the firm must adapt his prices to the asymmetric services. We report the results for the case $\delta = 0.6$ in Table 7.

We find that our main results in the base model of symmetric services qualitatively carry through to this extension. First, mixed bundling on usage remains dominantly more profitable than mixed bundling on variety. The best strategy is obtained under pay-per-use & subscription in most cases, and even in cases in which pay-per-use & subscription is not optimal (Beta(2,5), Beta(1,3), and Beta(3,1)), it can perform reasonably well. This suggests some robustness of the pay-per-use & subscription scheme to asymmetric services as well. Second, fixing mixed bundling on variety, whether one should use pay-per-use or subscriptions depends on the entire valuation distributions and there are cases in which either strategy can dominate the other. Third, pure bundling on variety is generally not recommended under pay-per-use, but can be a viable option when jointly used with subscriptions.

6.4 *Heterogenous Diminishing Rates of Consumption*

Our main model assumes homogeneous diminishing rates of consumption across customers. While this is in line with most existing multi-unit demand models (e.g., Goić et al., 2011; Jerath and Zhang, 2010), in reality customers may differ from one another not only in their valuations but also in their sensitivities to repeated consumption. We thus consider an extension in which a λ fraction of customers have a diminishing rate of 1 (as in our main model), and the remaining $1 - \lambda$ faction has a diminishing rate of $\gamma > 1$. For these latter customers, their marginal utility of consuming service i for $i = 1, 2$, given intrinsic valuation θ_i and past consumption n_i, is $\theta_i - \gamma n_i$. We assume that the intrinsic valuations θ_i have the same distributions across two customer types.

This extension captures the fact that in reality not all customers with high valuations of a service will consume this service more than those with low valuations, driven possibly by realistic factors such as time budgets and variety-seeking behaviors. Modeling these factors is beyond the scope of this extension and we leave a formal analysis of these factors to future research.

Although customers' purchase behaviors are differentiated by their diminishing rates, the formulations of these behaviors are similar to those in the main model. We omit these formulations for brevity and only report the revenue results in Table 8 for the case $\lambda = 0.5$ and $\gamma = 1.5$.

Once again, we find that the key results in our main model qualitatively carry over to this extension. Mixed bundling on usage is generally more profitable than mixed bundling on variety. Pure bundling on variety is suboptimal in all cases under pay-per-use, but can be profitable in all cases under subscriptions and in most cases under pay-per-use & subscription.

7 Nonlinear Pricing

Thus far our analysis of the pay-per-use scheme has focused on linear pricing. Under linear pricing, the fee charged per use of a service or service bundle is fixed irrespective of a customer's past purchase. The other usage-based pricing scheme, subscriptions, charges a fixed fee regardless of a customer's actual consumption. With flat fees or linear rates, these two pricing schemes both have seeming drawbacks. The former overlooks customers' diminishing margins of consumption and the latter fails to exploit customers' heterogeneous valuations. As a special case of nonlinear pricing, our proposed pay-per-use & subscription scheme in Section 5.1 partially alleviates these drawbacks and leads to substantial revenue improvement in some circumstances. This motivates us to study nonlinear pricing on a more granular scale.

Formally, nonlinear pricing corresponds to a menu of price-and-quantity quotations that differentiate customers based on their types (in our case, valuations). From a practical point of view, it can be implemented by extending the pay-per-use scheme using prices contingent on a customer's past purchase. To simplify analysis, we follow our main model (i.e., symmetric services and homogeneous diminishing rates of consumption) to study nonlinear pricing.

In general, multi-product nonlinear pricing is an extremely challenging problem as customers are heterogeneous in multiple dimensions (Rochet and Choné, 1998; see also our discussion in Section 7.2). Noting this challenge, we use a partial analysis of component selling and pure bundling to demonstrate the applicability of nonlinear pricing to settings with diminishing margins of consumption.

7.1 Component Selling Under Nonlinear Pay-Per-Use

With component selling, we formulate the firm's nonlinear pricing problem as a standard mechanism design problem. The firm decides a price-and-quantity menu $(n(\theta), p(\theta))$ for each service, where we suppress subscript i, for $i = 1, 2$ to simplify notations due to the symmetry of two services. We refer to a customer with intrinsic valuation θ as a θ-type customer. By revelation principle, it suffices to consider direct mechanisms under which customers truthfully reveal their types: a θ-type customer will *purchase* $n(\theta)$ units of a service and pay $p(\theta)$ accordingly. A subtlety in our analysis is that a θ-type customer, although purchasing $n(\theta)$ units, may not consume them all: she will selectively consume the service up to her satiation point. In other words, it is possible that a customer's purchase and consumption decisions are not identical under nonlinear pricing (recall that these quantities are always identical under component selling and linear pay-per-use).

Formally, define

$$U_\theta(\tilde{\theta}) := \int_0^{n(\tilde{\theta})} (\theta - w)^+ dw = \begin{cases} \theta n(\tilde{\theta}) - n(\tilde{\theta})^2/2, & \text{if } n(\tilde{\theta}) \leq \theta, \\ \theta^2/2, & \text{if } n(\tilde{\theta}) > \theta, \end{cases}$$

as a θ-type customer's aggregate utility of consumption if she purchases $n(\tilde{\theta})$ units of a service. This utility has a similar functional form as the one by Chellappa and Mehra (2018) but with a key difference. Specifically, a customer's consumption is bounded by her satiation point in our model, whereas it can be unbounded by Chellappa and Mehra (2018).[10] We formulate the firm's mechanism design problem as follows:

PROBLEM 1.

$$\max_{n(\cdot),\, p(\cdot)} \quad 2\int_\theta p(\theta) f(\theta) d\theta$$
$$s.t. \quad U_\theta(\theta) - p(\theta) \geq U_\theta(\tilde{\theta}) - p(\tilde{\theta}) \text{ for all } \tilde{\theta},$$
$$U_\theta(\theta) - p(\theta) \geq 0.$$

The satiation point is a critical feature that differentiates Problem 1 from Chellappa and Mehra (2018). However, we establish in the following result that its presence does not alter a firm's optimal menu $(n(\cdot), p(\cdot))$, as in optimality, a customer's satiation point is never reached.

PROPOSITION 8. *Suppose* $\theta_i \sim U(0, 1)$, $i \in \{1, 2\}$. *The firm's optimal revenue in Problem 1 is* $1/6$, *achieved under the following price-and-quantity menu:*

$$(n_i(\theta), p_i(\theta)) = \begin{cases} (2\theta - 1, -\theta^2 + 2\theta - 3/4), & \text{if } \theta \geq 1/2, \\ (0, 0), & \text{if } \theta < 1/2. \end{cases}$$

It follows from Proposition 8 that the optimal pay-per-use price of a service, given a customer's past purchase quantity n, is $(1 - n)/2$ under uniform valuations, which is decreasing in n. This is illustrative of how nonlinear pricing can be used to accommodate customers' diminishing margins of repeated consumption.

7.2 Pure Bundling Under Nonlinear Pay-Per-Use

We next study pure bundling under nonlinear pay-per-use. The firm decides a menu $(n_B(\boldsymbol{\theta}), p_B(\boldsymbol{\theta}))$, where $\boldsymbol{\theta} := (\theta_1, \theta_2)$, to sell the service bundle. Define

$$U_{\boldsymbol{\theta}}(\tilde{\boldsymbol{\theta}}) := \int_0^{n_B(\tilde{\boldsymbol{\theta}})} [(\theta_1 - w)^+ + (\theta_2 - w)^+] dw$$

as a $\boldsymbol{\theta}$-type customer's aggregate utility of consumption if she purchases $n(\tilde{\boldsymbol{\theta}})$ units of the bundle, where we have used the fact that the customer will consume each service only up to her satiation point. In this case, it is possible that the consumption of *both* services is strictly less than the purchase quantity (recall from Lemma 1 that a customer's consumption of at least one service should match her purchase quantity under pure bundling and linear pay-per-use).

We formulate the firm's mechanism design problem under pure bundling as follows.

PROBLEM 2.

$$\max_{n_B(\cdot),\, p_B(\cdot)} \int_{\theta} p_B(\boldsymbol{\theta}) f_{1,2}(\boldsymbol{\theta}) d\boldsymbol{\theta}$$
$$s.t. \quad U_{\theta}(\boldsymbol{\theta}) - p_B(\boldsymbol{\theta}) \geq U_{\theta}(\tilde{\boldsymbol{\theta}}) - p_B(\tilde{\boldsymbol{\theta}}) \text{ for all } \tilde{\boldsymbol{\theta}},$$
$$U_{\theta}(\boldsymbol{\theta}) - p_B(\boldsymbol{\theta}) \geq 0,$$

where $f_{1,2}$ denotes the joint pdf of $\boldsymbol{\theta}$.

Despite a similar formulation, Problem 2 that optimizes over the price-and-quantity menus for the service bundle is fundamentally different from Problem 1, and more broadly, the classic mechanism design problems. This fundamental difference lies in the fact that a customer's type (corresponding to her valuations) in Problem 2 is two-dimensional whereas it is one-dimensional in Problem 1. There are two solution approaches developed for one-dimensional problems in the mechanism design literature, namely, *parametric-utility* and *demand-profile*. However, extending these two approaches to multi-dimensional problems is extremely challenging (see Rochet and Stole, 2003 and the references therein). As a result, most multi-dimensional mechanism design problems do not afford a tractable characterization except for a few special cases (Laffont et al., 1987; Armstrong, 1996; Rochet and Choné, 1998 are the known exceptions to date). Regarding these two approaches, the parametric-utility approach is more computationally intensive even for one-dimensional problems and there is no clear path of extending it to multi-dimensional problems. The demand-profile approach has a simple implementation and can extend to multi-dimensional problems, but applying this approach requires the price function derived from this approach to cut each customer's demand curve at most once. Our computation shows that this ad-hoc requirement actually fails when we apply this approach to solve Problem 2, so it does not work for our purpose either. We provide more details in Appendix C in the E-Companion of why this approach fails.

Therefore, we are unable to derive the optimal solution to the two-dimensional mechanism design Problem 2. Nevertheless, motivated by the optimal price under component selling and nonlinear pay-per-use (cf. Proposition 8), we consider a price function under pure bundling in the form of $s(w) = (\alpha - \beta w)^+$ with $\alpha, \beta \geq 0$, such that a customer who has purchased n_B units of the bundle is asked to pay a rate $s(w)$ for an additional purchase of the bundle. So, the total price paid to purchase n_B units of the bundle is $p(n_B) = \int_0^{n_B} s(w)dw$. If $\alpha \geq \beta$, then $s(w) > 0$ for all $w < 1$, that is, the firm charges a positive price for every use of the bundle. In this case, one can show that this $s(w)$ will cut each customer's demand curve $(\theta_1 - w)^+ + (\theta_2 - w)^+$ at most once for all θ_1 and θ_2. This allows us to apply the demand-profile approach (see Wilson, 1993; Rochet and Stole, 2003) and specify the firm's revenue in a simple form. If $\alpha < \beta$, then customers who have purchased α/β units of the bundle can have unlimited access. In this case, the single-cut requirement between $s(w)$ and customers' demand curves is violated. This suggests a more complicated form of the firm's revenue that cannot be simplified, so we can only numerically compute the optimal pay-per-use price $s(w)$.

Table 9. Revenue comparison under nonlinear pay-per-use: uniform valuations, $s(w) = (\alpha - \beta w)^+$ under pure bundling.

	Revenue	$s(w)$
Component selling	0.167	
Pure bundling ($\alpha \geq \beta$)	+0.3%	$0.7 - 0.7w$
Pure bundling (general α and β)	+1.2%	$(0.742 - 0.816w)^+$

PROPOSITION 9. *Suppose $\theta_i \sim U(0,1)$, $i \in \{1,2\}$. Under nonlinear pay-per-use, the firm's revenue is strictly higher under pure bundling than under component selling.*

In the proof of Proposition 9, we focus on uniform valuations and find a feasible solution of $s(w)$ under which the firm's revenue is strictly higher than the optimal revenue under component selling. We numerically compute the optimal pay-per-use prices $s(w) = (\alpha - \beta w)^+$ under pure bundling with and without the constraint $\alpha \geq \beta$, and present them in Table 9.

Proposition 9, together with Table 9, shows the following important result. Pure bundling on variety, which we showed is suboptimal under linear pay-per-use, turns out to be a profitable strategy under nonlinear pay-per-use. As we demonstrate shortly, this is a robust result that extends to general non-uniform valuation distributions. This suggests that variety bundling, jointly with nonlinear pricing, can be an efficient instrument to accommodate customers' diminishing margins of consumption. To understand this result, note that nonlinear pricing allows the firm to alleviate the compensation effect by charging pay-per-use prices that decrease with a customer's past purchase quantity. In this way, a customer's (dis)utility of purchasing her less preferred service in the bundle at average price $s(w)/2$ does not have to be sufficiently negative and can be easily compensated by the utility of purchasing her preferred service. Then, with a mitigated compensation effect, the reduced valuation dispersion rooted in bundling restores its power and renders bundling more profitable.

It is significant that the revenues under nonlinear pay-per-use (0.169, achieved under pure bundling and nonlinear pay-per-use) are higher than the highest revenue obtained among all pricing strategies with flat fees or linear rates (0.164, achieved under pure bundling and pay-per-use & subscription). This is expected because nonlinear pricing in this section is considered on a much more granular scale than other simple schemes with flat fees or linear rates. However, implementing nonlinear pricing requires managing a continuum of prices and keeping track of each customer's purchase history. Thus, one should tradeoff the economic benefits of nonlinear pricing against the increased operational costs (which we don't model in this article) that arise due to its sophistication.

Table 10. Revenue comparison under nonlinear pay-per-use: beta distributions.

Beta distribution	Optimal[a] flat fee	Component selling pay-per-use	Pure bundling pay-per-use
Beta(2,2)	0.149	−1.4%	+3.1%
Beta(2,5)	0.048	−1.0%	+4.6%
Beta(1,3)	0.043	−0.8%	+3.1%
Beta(3,1)	0.359	−1.8%	+2.1%

"Optimal Flat Fee[a]" refers to the optimal strategy among mixed bundling under pay-per-use, mixed bundling under subscriptions, component selling under pay-per-use and subscription, and pure bundling under pay-per-use and subscription, whichever achieves the highest revenue.

Non-Uniform Valuation Distributions. We next consider nonlinear pricing under non-uniform valuations. Following the convention in the mechanism design literature, we focus on beta distributions with increasing hazard rates. (Using this criterion we rule out Beta(0.5, 0.5) in our analysis.) To compute the optimal revenue under component selling, we first follow the proof of Proposition 8 and establish that customers' satiation points do not affect the firm's optimal mechanisms, and then apply the approach by Chellappa and Mehra (2018) to compute the firm's optimal pay-per-use prices. Table 10 reports our computational results. The revenues under component selling are precise numbers obtained from this procedure and thus represent the *true optimal* revenues. We are unable to derive the optimal pay-per-use prices under pure bundling for reasons elaborated previously. Instead, we choose to consider pay-per-use price functions in the form of $s(w) = (\alpha - \beta w)^+$. In this sense, the revenues under pure bundling presented in Table 10 provide a lower bound of the true optimal ones. Despite being a lower bound, these revenues under pure bundling unambiguously dominate the true optimal revenues under component selling, as well as those with flat fees and linear rates. These observations are consistent with our main findings under uniform valuations.

8 Conclusion

Product bundling and subscription pricing are both widely used in the service industry, each catering to customers' desires in one dimension of variety and usage. Despite their prevalence, the joint effect of these two pricing strategies is not well understood in multi-service settings. To fill in this gap, we developed a multi-unit demand model that accounts for customers' diminishing margins of consumption, and investigated pricing strategies that span both variety and usage.

Our results provide important insights. First, we showed that, due to a newly identfied compensation effect, pure bundling results in lower revenues than component selling under the pay-per-use scheme. However, pure bundling regains its superiority over component selling under subscriptions. We illustrated that mixed bundling on variety can effectively mitigate the compensation effect and is generally more profitable than both component selling and pure bundling. This may help explain why Disney employs multiple subscription options, such as Hulu, Disney+, and ESPN+, both individually and in bundled packages, to manage its streaming services. Nevertheless, the decision to use pay-per-use or subscriptions on usage in conjunction with mixed bundling on variety depends on the tail distributions of customers' valuations. Our study generated instances in which either strategy can outperform the other.

We also examined the combination of pay-per-use and subscriptions, which resembles applying mixed bundling to the usage dimension. Our analysis suggested that pure bundling on variety, jointly with pay-per-use & subscription, can often perform best. Further, with fixed component selling on variety, both the optimal pay-per-use price and subscription fee under the pay-per-use & subscription scheme are higher than their counterparts when only one of pay-per-use and subscriptions is adopted in the usage dimension. We found that even in cases in which this strategy is not optimal, it can perform reasonably well too. Thus, we advocate mixed bundling on usage (i.e., pay-per-use & subscription) for the general principle. This partially explains why most amusement parks choose to offer a combination of annual and daily passes.

Our key results can extend to cases of correlated valuations of services, asymmetric services with vertical differentiation, and customers' heterogeneous diminishing rates of consumption. Mixed bundling on usage continues to be more profitable than mixed bundling on variety in these settings. We also studied multi-product nonlinear pricing, and showed that pure bundling on variety can outperform component selling under nonlinear pay-per-use. In other words, bundling on variety, combined with nonlinear pricing, can effectively accommodate customers' diminishing margins of repeated consumption.

Our study emphasizes the importance of customer demand models, particularly the distinction between single-unit and multi-unit demands with diminishing margins for repeated consumption, in driving monopoly firms' strategic choice of product bundling. Our findings suggest caution for monopoly firms that unambiguously practice product bundling in search of better sales without optimizing this decision jointly with their pricing decisions on usage.

We acknowledge that there are other realistic factors not captured in our model such as competition (Armstrong and Vickers, 2010), buyers' demand uncertainty (Png and Wang, 2010), and time budgets. These factors, if present, will also critically affect the profitability of product bundling as well as the efficacy of pay-per-use and subscriptions. Extending our framework to incorporate these factors will be valuable endeavors for future research. We hope our work will invite more investigations into this exciting strand of research.

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Notes

1. For example, Ocean Park Hong Kong operates a regular park and a water-themed park, each selling its own tickets. Visitors with a daily pass cannot reenter a park after exiting from it, whereas visitors with an annual pass can make unlimited entry attempts throughout the year. We thus consider the one-time entry to one park as a single service in this setting.
2. In many service settings, staff are salaried and facility costs are fixed, so it is reasonable to assume negligible marginal costs of service provision (e.g., Cachon and Feldman, 2011).
3. Note that without diminishing margins, that is, each unit of consumption yields identical utility, customers under linear pay-per-use (applicable to both component selling and pure bundling) will purchase either zero or a maximal quantity. This effectively reduces to the classical single-unit demand framework.
4. We assume that customers, upon purchase of a bundle, have sufficient time budgets to consume both services provided that they are willing to do so.
5. Contexts where mixed bundling reduces to component selling or pure bundling can be found by Prasad et al. (2010); Wu et al. (2022); Jin et al. (2022) with new defining features such as network externality, valuation uncertainty, and competition.
6. Universal Orlando does not offer "1-Park Annual Pass," that is, it does not adopt mixed bundling to sell subscriptions; see https://www.universalorlando.com/web/en/us/tickets-packages/annual-passes/fl-resident-prices#2-park-annual-passes.
7. For example, when considering the single-unit demand model under uniform valuations, the optimal revenues under pure bundling and mixed bundling in a two-product setting are $2\sqrt{6}/9 \approx 0.544$ and $(12 + 2\sqrt{2})/27 \approx 0.549$, respectively (Jin et al., 2022). Thus, the revenue improvement of mixed bundling over pure bundling in this case is roughly 0.9%. See also Table 3 for illustrations of revenue improvement of mixed bundling over pure bundling under other valuation distributions in the single-unit demand model.
8. A similar strategy is considered by Sundararajan (2004) for information goods. However, Sundararajan (2004) assumes that customers' demands for an information product can be unbounded.
9. Allowing mixed bundling on variety will require four prices in implementation (two for pay-per-use and two for subscriptions). To facilitate a fair comparison with all other two-price schemes discussed in this article, we choose not to consider mixed bundling and only focus on component selling and pure bundling on variety.
10. The utility function by Chellappa and Mehra (2018) is quadratic in $n(\tilde{\theta})$ for all $n(\tilde{\theta}) \geq 0$.

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